9 *Conclusions and Recommendations*

> Plato asks us to imagine an underground cave which has an opening towards the light. In this cave are living human beings, with their legs and necks chained from childhood in such a way that they face the inside wall of the cave and have never seen the light of sun. Above and behind them, i.e. between the prisoners and the mouth of the cave, is a fire, and between them and the fire is a raised way and a low wall, like a screen. Along this raised way there pass men carrying statues and figures of animals and other objects, in such a manner that the objects they carry appear over the top of the low wall or screen. The prisoners, facing the inside of the wall of the cave, cannot see one another nor the objects carried behind them, but they see the shadows of themselves and of these objects thrown on to the wall they are facing. They see only shadows.
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> Plato's allegory of the Cave, described by
> Frederick Copleston

9.1 Summary

This dissertation implemented advanced signal processing in the analysis of seismic surface wave propagation. The dissertation strove to merge two separate engineering fields to allow significant advances to the traditional, simplistic geotechnical spatial signal processing methods. The application of advanced digital signal processing and spatial array processing yielded multimodal phase velocity and attenuation estimates and solidified the underlying theory controlling resolution and aliasing criteria.

9.2 Conclusions

Traditional analysis of seismic surface waves suffers from several limitations. In some cases, the limitations have been long observed in experimental data, while in other cases, the limitations have not fully been recognized. This dissertation reviewed the major limitations of the traditional phase velocity and attenuation estimators, determined their underlying causes, and offered improved and optimum solutions. Passive source surface wave analysis was given better perspective in spatial array processing, analyzed with advanced spectrum estimators, and extended to yield passive attenuation coefficients.

9.2.1 Active Phase Velocity Estimation Limitations and Solutions

The limitations of traditional active phase velocity estimators stem from two primary areas: 1.) Simplistic signal processing methods, and 2.) A model incompatibility. The traditional two-point estimators rely on poor spectral characteristics, do not solve the phase velocity estimation problem in any optimum fashion, and rely on subjective data inclusion decisions. The estimation of an average wavenumber from two experimental samples corresponds to attempting to estimate temporal frequency content from only two temporal samples. In multimodal wavefields, the traditional cross power spectrum method may estimate negative velocities depending on the sensor locations. The traditional transfer function method velocity estimate tends to zero as frequency tends to zero regardless of the site characteristics.

Near-field effects represent the most often encountered impediment in geotechnical active seismic surface wave analysis. The near-field effects were shown to consist of three unrelated phenomenon. A model incompatibility, due to modeling a cylindrically spreading wavefield with a plane wave estimator, introduced the most significant error into phase velocity estimation. The model incompatibility effects on phase velocity were theoretically analyzed with the correct cylindrical wave equation solution. The common recommendations for mitigation of near-field effects correspond to the acceptable errors expected in phase velocity estimates due to the model incompatibility. Cylindrical beamformers allowed cylindrical wavenumbers and phase velocity estimates to be obtained from experimental data.

Traditional estimators lack the ability to estimate single mode phase velocities or determine which mode or mix of modes is contained in the estimate. The inability to handle multiple modes has led to several traditional phase velocity definitions, including apparent and effective phase velocity. Rather than corresponding to a true material or soil profile property, i.e. an eigenvalue, the traditional velocity definitions are a function of the position of the measurements. The conventional and adaptive spatial array estimators handle multiple modes, allowing the estimation of a multimodal dispersion curve, and the phase velocity estimates correspond to a material property, i.e. the phase velocity is not a function of the position of the measurement. Instead, the advanced phase velocity estimators rely on the spatial filter created by the sampling characteristics and sensor weight vector.

9.2.2 Improvements and Extensions to Active Phase Velocity Estimation

Several improvements and extensions to the traditional active source seismic surface wave phase velocity estimation problem were introduced. The theory of random processes, along with assumptions regarding the noise power characteristics, allowed the traditional two sensor experimental measurement procedure to be recast into a synthetic linear array problem. The ability to isolate single modes and resolve multiple signals relies on the creation of a synthetic array with improved spatial spectral properties compared to the traditional two-sensor problem.

Introduction of the correct Hankel function solution of the cylindrical wave equation vastly improved the analysis of phase velocity estimation procedures and allows phase velocities to be estimated for larger wavelengths compared to previous analysis methods. The correct cylindrical model allowed a detailed study of the model incompatibility effects on traditional phase velocity estimates. The conventional and adaptive signal processing methods allowed the estimation of multimodal phase velocities and power ratios.

9.2.3 Active Attenuation Estimation Limitations and Solutions

The major limitations of attenuation estimation methods were discussed. The inability to remove geometric spreading represents the greatest traditional impediment to determining material attenuation. The traditional $\sqrt{x}$ model was shown to yield biased and non-conservative attenuation estimates. The minimization of noise effects represents an additional limitation of traditional estimators. Two traditional methods of noise removal have been implemented. Although both methods work in a stationary noise field, they only account for ambient seismic noise, and therefore, do not filter competing signals. Spatial array processing and the eigenvalue extremal property of power spectrum estimation theory yielded advanced methods to minimize the effects of competing signals and to remove stationary noise.

9.2.4 Active Attenuation Estimation Extensions and Improvements

Implementation of the correct Hankel function cylindrical wave equation solution allowed determination of the correct cylindrical geometric spreading function. Geometric spreading of energy was shown to be a function of wavenumber in the Hankel function solution, and the complete spectral representation for geometric spreading $G(\omega,\mathbf{k})$ was derived. Since the natural wavenumbers are site-dependent, the decay of energy due to geometric spreading is also site-dependent. The wavenumbers of a cylindrical wavefield control geometric attenuation, velocity of propagation, and the spacing of zeros, which follow a Bessel function solution. Multimodal attenuation curves were estimated.

9.2.5 Passive Method Improvements and Extensions

Previously implemented passive surface wave analysis methods relied on insufficient array processing theory. Prior aliasing and resolution characteristics relied on either active source recommendations, which do not represent the plane wave, far-field passive problem, or empirical comparisons between the results yielded by arrays of different sizes at the same site. The theory of spatial array processing, including aliasing and resolution criteria, was placed in proper perspective.

Single mode phase velocity estimates were obtained with conventional and adaptive spectrum estimation methods. MUSIC, which only considers the noise subspace, yielded the narrowest signal related peaks and smoothest background power estimate.

Two methods for obtaining passive attenuation coefficients were presented. Least-squares fitting a plane to displacement magnitudes yielded attenuation coefficients corresponding to the lowest wavenumber, and therefore, longest wavelength, contained in the passive wavefield. The least-squares fitting of a plane suffers from an inability to filter competing signals, although ambient seismic noise was removed with the smallest eigenvalue of the spatiospectral correlation matrix. The orthogonal projection sub-array method allows the filtering of competing signals and offers the greatest potential to obtain individual modal and multimodal passive attenuation coefficients. Signal modeling of experimental measurements provided an excellent fit of experimental measurements with the estimated wavenumber and attenuation coefficient.

9.3 Recommendations

The methods presented in this dissertation will yield improved estimates with larger array and synthetic array dimensions and more samples. For active surface wave methods, obtaining an extra sample has been simplified with the derivation of synthetic linear arrays. Obtaining more passive samples requires the addition of an extra channel and sensor. Using larger array geometries, with greater maximum spatial lags, will yield resolution increases. Using a greater number of sensors will increase the computational requirements, since the size of the matrices and matrix inversions will also increase. The size and location of the site being tested may dictate which test may be utilized, since some test sites may not be large enough to allow a large two-dimensional spatial array to be deployed, while other sites near traffic or in urban environments may be too noisy to implement active tests. The recommendations for future research can be broken into synthetic studies, practical improvements to estimators and speed of implementation, and theoretical and long-term algorithm development.

9.3.1 Practical Considerations

The active, linear array estimators allow significant computational savings with the implementation of the Fast Fourier Transform. After placing the non-uniformly spaced samples onto a uniform sampling grid, the FFT allows the entire dispersion curve to be estimated as the diagonal of a two-dimensional matrix, as discussed in Appendix A. In the case of the two-dimensional spatial array, implementation of the FFT in a similar procedure requires computing Fourier transforms along three-dimensions and creates a two-dimensional matrix in which all the entries yield physically meaningful estimates. The extension to the third dimension significantly increases computation. In this dissertation, the two-dimensional, passive source spatial array spectrum estimates always used discrete sampling grids of the wavenumber domain, implemented without efficient use of the FFT. Therefore, an important improvement to the implementation of the spectrum estimators for two-dimensional, plane wave sources would be the use of algorithms to increase speed of computation.

An additional improvement would be the ability to optimally calculate only a portion of the wavenumber plane spectrum, analogous to the chirp transform in one-dimensional signal processing. The efficient algorithm could then be used to estimate the wavenumber power spectrum at a finer spacing, which would yield better estimates for both phase velocity and attenuation.

9.3.2 Synthetic and Theoretical Studies

Two studies would significantly improve the understanding of seismic surface waves in geotechnical engineering. First, explicitly determining the profile-dependent amount of body wave reflections that change mode into surface waves upon reflection would allow additional insight into the modal propagation of Rayleigh surface waves.

Second, studies of the statistical and stationarity properties of passive sources typically used in geotechnical engineering will aid in future analyses. Passive sources, although exhibiting far-field, plane wave behavior, offer several impediments to the estimation of engineering parameters. The temporal statistics of the wavefield may be stationary over only a limited time, which affects the applicability of the power spectrum estimators. The spatial position of some passive seismic sources may also exhibit limited stationarity. If the position or range of the source changes during the measurement, the performance of the power spectrum estimators may deteriorate due to perceiving the moving source as more than one signal.

9.3.3 Theoretical and Long-Term Algorithm Development

The long-term algorithm and theoretical development of analysis of seismic surface waves primarily deals with creating inversion algorithms that can utilize the information yielded by advanced signal processing methods. The fundamental mode inversion is now questionable considering the estimated multimodal dispersion relations, which show the fundamental mode does not dominate as previously assumed. The three-dimensional inversion procedures, which synthetically recreate the active test with a correct cylindrically spreading Green's functions, are no longer necessary if the experimental wavenumber estimates use the correct cylindrical model. The three-dimensional inversion techniques also suffer from numerically expensive computation procedures. The ability to isolate individual modes and multiple modes allows reconsideration of the traditional inversion procedures. Electrical engineering linear systems theory and adaptive filters hold great promise if applied properly to the geotechnical inversion problem.

An additional category of filters are called *dynamic-adaptive* (Johnson and Dudgeon, 1993). Dynamic-adaptive filters update the filter and spatiospectral correlation matrix as a function of time, as well as a function of noise and signal characteristics. The dynamic-adaptive structures are advantageous in nonstationary signal environments and where the solution space varies with time. With increases in the capabilities of signal analyzing equipment, the dynamic-adaptive methods can be used in geotechnical engineering for real-time tracking of signal sources and solution spaces. As a more practical consideration, since typical geotechnical problems do not require real-time tracking, the dynamic-adaptive methods will offer improvements in data analysis after data collection. Real-time analysis of geotechnical problems is also limited by the necessity of calculating frequency-wavenumber power spectrums for a large range of frequencies.

Different array geometries should be implemented for the estimation of passive attenuation coefficients. Use of sub-array designs, such as three five-sensor circular arrays spaced on a triangular grid, offer an alternative method to estimate passive attenuation coefficients.