2 *Traditional Seismic Surface Wave Tests*

It is proposed to investigate the behavior of waves upon the plane free surface of an infinite homogeneous isotropic elastic solid, their character being such that the disturbance is confined to a superficial region, of thickness comparable with the wavelength.

John William Strutt, Third Baron Rayleigh,
"On Waves Propagated Along the Plane
Surface of an Elastic Solid", 1885

2.1 Introduction

Spectral analysis of surface waves (SASW) tests were developed in the early 1980s (Stokoe et al., 1989; Tokimatsu, 1995) as an extension of the Steady State Vibration Test (Richart, et al., 1970). The typical experimental and data processing procedures have made several advancements and improvements through the past 15 years, and the test has been extended to handle passive energy sources. Although Love surface waves have been used to aid in the determination of engineering parameters (Tokimatsu, 1995), SASW tests primarily rely upon the propagation of Rayleigh surface waves. Rayleigh surface waves will always be the subject of discussion in this dissertation unless otherwise noted.

Surface wave methods offer several advantages over traditional lab and in situ methods used in geotechnical engineering. The tests are noninvasive, and therefore, do not require the use of boreholes. The noninvasive nature is especially advantageous when testing environmentally sensitive materials, such as landfill wastes (Rix and Lai, 1998). The material is tested in situ, avoiding disturbance and handling difficulties encountered in lab tests. The tests also cover a large spatial area, yielding a more average representation of the underlying soil profile.

Initial surface wave methods could only estimate shear wave velocity profiles of soils or pavements. The test was extended to estimate the damping ratio profile with a separate experimental measurement procedure. Recently, the tests have been combined to estimate attenuation and phase velocity from a single measurement procedure, and the inversion algorithms have been generalized to simultaneously estimate a damping ratio and shear wave velocity profile (Lai, 1998; Rix and Lai, 1998).

Traditional SASW data analysis relies on limited spatial samples and elementary signal processing. Although most layered profiles exhibit multiple modes of propagation, the traditional analysis procedure is unable to distinguish between different modes. Additionally, the current signal processing methods do not explicitly consider interfering noise, the underlying theory of random processes, and the consequences of spectral domain operations on experimental estimates.

The chapter begins with a review of the general theory of seismic surface wave propagation. Engineering analysis of seismic surface waves consists of two problems. First, the experimental dispersion and attenuation curves must be estimated from seismic surface wave measurements. Second, the dispersion and attenuation curves are used as the input for an inversion problem to determine the dynamic engineering properties of the layered soil profile. After covering the traditional spectral analysis of surface waves test setups, the traditional dispersion and attenuation curve estimators are discussed. To introduce the complete surface wave analysis problem, the inversion problem is also briefly discussed. Finally, a critical analysis of the major limitations of traditional surface wave analysis is given. The limitations and problems identified serve as the motivation for the discussion and results presented in the remainder of the dissertation.

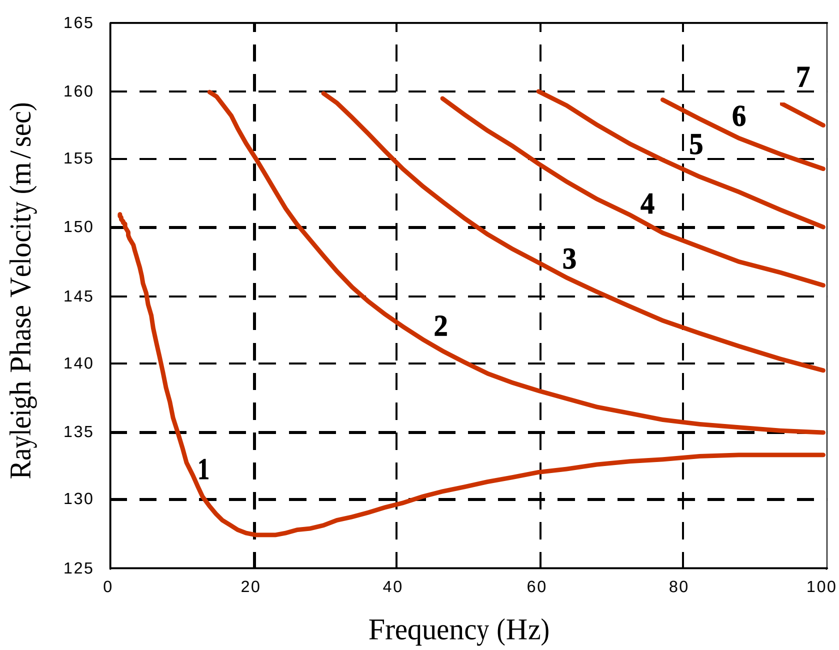

Figure 2.1 Multiple Mode Dispersion Relationship for Vertically Heterogeneous Media (From Lai, 1998)

2.2 Seismic Surface Waves and Dispersion

In a homogeneous elastic half-space with no damping, Rayleigh surface waves propagate along the surface of the body at a constant phase velocity, i.e. different frequencies propagate with the same velocity. In a layered or vertically heterogeneous half-space, such as an ideally layered soil profile, different frequency surface waves propagate with different phase velocities and different wavelengths. The relationship between phase velocity, frequency, and wavenumber is called the dispersion relation. The Rayleigh wave propagation problem constitutes an eigenproblem, where the wavenumbers are the eigenvalues for the linear (at small strains), shift invariant soil system, as shown in Figure 1.3. Figure 2.1 shows an idealized example of a multimodal dispersion relation, and Figure 2.2 shows the displacement eigenfunctions as a function of depth due to the different modes. As seen in Figure 2.1, lower frequency waves tend to propagate with higher velocities and several modes may exist in a single soil profile. In addition, higher modes propagate with longer wavelengths and sample the soil profile to larger depths, as seen in Figure 2.2.

The primary desired functions from SASW tests are the dispersion and attenuation curves. Several physical and environmental phenomena interfere with test analysis, including the possible presence of multiple modes, interfering body waves produced by the source, and interfering background or ambient noise energy.

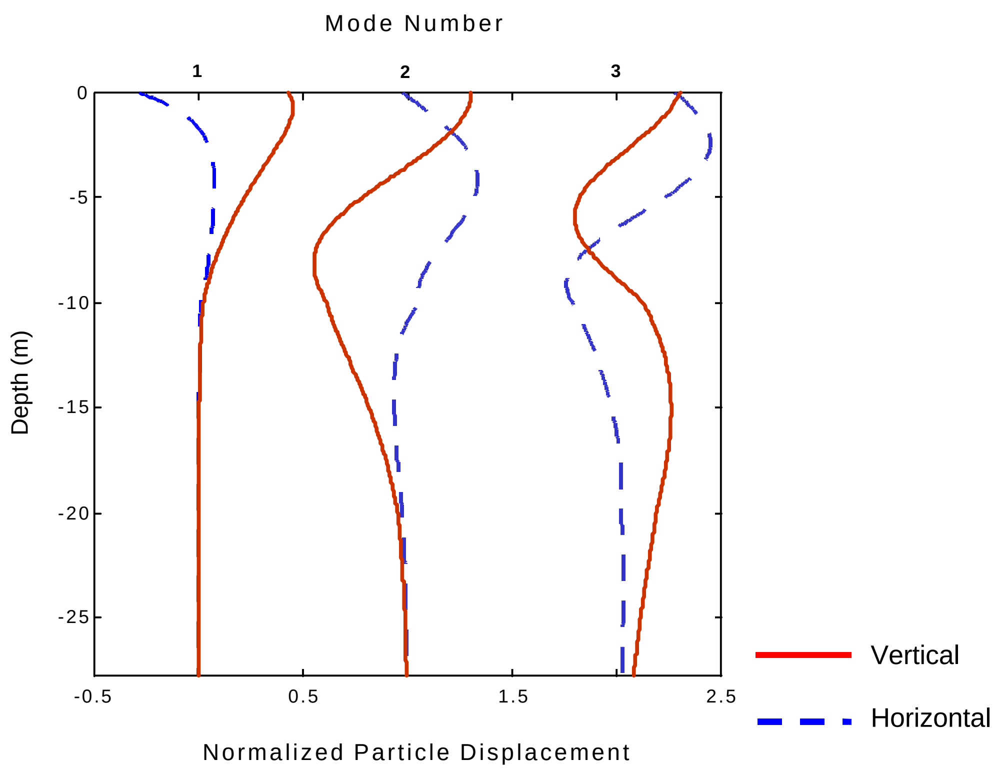

Figure 2.2 Rayleigh Wave Displacement Eigenfunctions in Vertically Heterogeneous Media (From Lai, 1998)

2.3 Spectral Analysis of Surface Waves (SASW) Test Setup

Several experimental measurement source-receiver arrangements utilizing two sensors and an active source have been implemented. The traditional setups have been recommended due to their ability to mitigate near-field effects and match theoretical and numerical modeling expectations (Sanchez-Salinero, 1987).

Spatial array measurements of passive sources, e.g. microtremors and cultural noise, recently have increased the ability to estimate phase velocity at lower frequencies and longer wavelengths. Two-dimensional and linear spatial arrays enable the use of more advanced signal processing techniques due to the simultaneous measurement of the wavefield at all sensors. Since the sensors all measure the same ambient wavefield, the advanced signal processing methods can explicitly account for the noise and improve the resolution and statistical properties of the phase velocity and attenuation estimates.

2.3.1 Traditional Cross Power Spectrum Two-Point Method

The traditional active test setup uses two sensors to measure an actively produced Rayleigh surface wavefield at several spatial lags. The precursor to the SASW test was implemented by manually moving two sensors along the ground surface until the sensor displacements were in phase, and therefore, a phase velocity could be estimated (Richart, et al., 1970). This original testing procedure was very time consuming and cumbersome to implement, and, additionally, offered no ability to consider multiple modes of propagation.

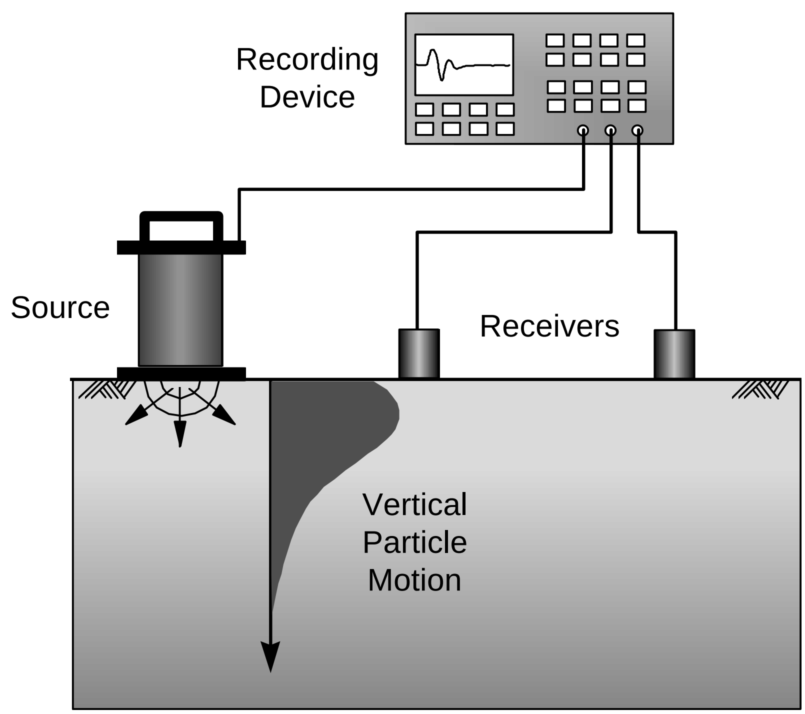

Figure 2.3 Traditional Active Cross Power Spectrum Two-Sensor SASW Test Setup (From Rix, 1997)

The most commonly used experimental sensor setup, called the Common Source Array, uses equal spatial separation between the source and first receiver and between the first and second receiver, as shown in Figure 2.3. The spatial lag distance is typically doubled for each successive measurement, and the phase as a function of distance is used to determine phase velocity. The increased distance between the first receiver and source decreases the detrimental near-field effects, traditionally thought to be caused by body wave interference (Sanchez-Salinero, 1987), but the changing location of the first receiver also makes it impossible to measure multiple modes or isolate a single mode. Another commonly used setup, the Common Receiver Midpoint Array, moves both the sensors and source for each spatial lag measurement, maintaining a constant midpoint between the two receivers (Lai, 1998).

The procedures work well for single modes of propagation, but in layered and vertically heterogeneous soil profiles multiple modes always exist. The inability to resolve multiple modes introduces error into the dispersion curve estimate, since multiple modes become superposed depending upon their participation in the wavefield.

2.3.2 Traditional Transfer Function Two-Point Method

A more recent active SASW experimental procedure places a sensor on the harmonic energy generator and varies the spatial lag between the source and a single receiver, as shown in Figure 2.4 (Sanchez-Salinero, 1987; Lai, 1998). The transfer function between the source and receiver is then used to determine the phase velocity and magnitude

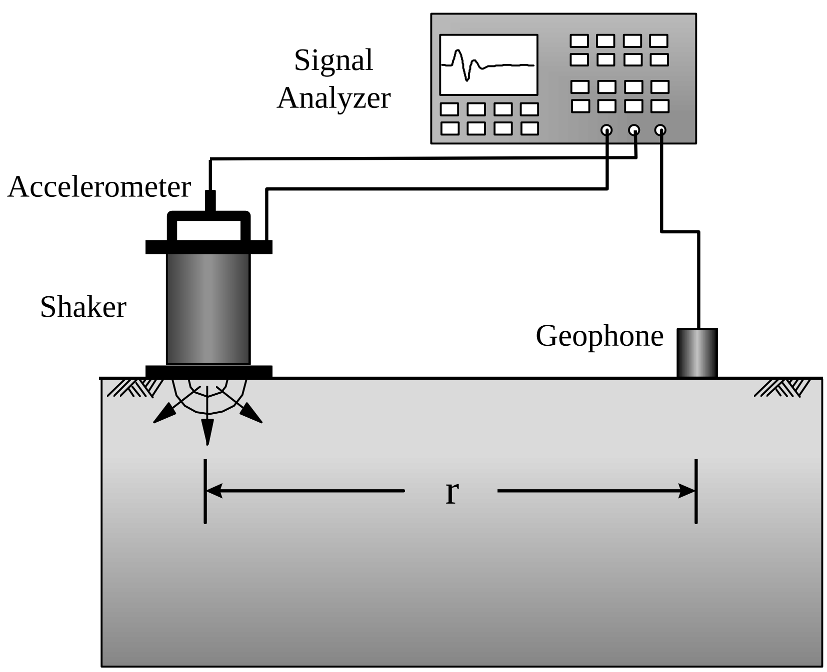

Figure 2.4 Traditional Active Transfer Function Two-Point SASW Test Setup (From Rix, 1997)

across a range of wavelengths. The test works well for a single mode of propagation, but the traditional transfer function method data analysis procedure is unable to resolve multiple modes, and therefore, suffers the same disabilities as other traditional SASW methods.

The transfer function method offers better attenuation estimates due to the stationary position of the first sensor. Previous attenuation estimation procedures based on doubling the receiver-source distance failed due to the presence of geometric spreading. The samples at various spatial lags could not be combined efficiently to estimate a single attenuation coefficient as a function of frequency. The transfer function testing procedure allows all the experimental measurements to be combined to estimate a single attenuation coefficient as a function of frequency because it establishes a reference point for all magnitude measurements and allows the wavefield model to introduce geometric spreading.

2.3.3 Linear Array Measurements

With the increased use of multichannel data acquisition systems, the most recent active SASW measurement procedure deploys a linear array of sensors. Linear arrays used in the geotechnical and geophysical fields have ranged from 5 or 6 sensors (Tokimatsu, 1995) to over 60 sensors (Park et al., 1999). Measurements from a greater number of sensors not only increase the ability to consider noise, but also allow a much greater range of spatial lags to be sampled in less time. The synthetic linear array introduced in Chapter 6 yields the increased analysis power of linear arrays without requiring the deployment of numerous sensors at the same time.

2.3.4 Spatial Array Measurements

Measurements of passive surface waves with two-dimensional spatial arrays represent the most recent addition to the SASW testing theory (Tokimatsu, 1995). Spatial arrays have a long history in electrical engineering and have recently been introduced to geotechnical engineering from the field of seismology (Horike, 1985). The measurement of passive waves requires a much larger number of sensors and greater coverage of spatial lags due to the necessity of isolating a two-dimensional vector direction of propagation. The measurement of passive surface waves has allowed the depth of engineering parameter estimation to increase considerably due to the longer wavelengths of typical passive sources.

The underlying theory of spatial array processing has not received a thorough treatment in the geotechnical literature. Detailed theoretical interpretations of resolution and array spectral operators exist for the passive dispersion curve estimation methods, but in the geotechnical literature, the theory behind the array operations have been glossed over while proceeding to the experimental results. Therefore, the future implementation of passive surface wave methods demands a detailed consideration of spatial aliasing, multimode propagation, possibility of multiple sources, and effect of nonstationary wavefields.

2.4 Dispersion Curve Estimation

The phase velocity as a function of frequency, or dispersion curve, is one of the primary functions of interest for SASW experimental measurements. Traditionally, the phase velocity is estimated over a range of two-point spatial lags, and the two-point estimates are averaged to estimate a single wavenumber as a function of frequency. Using spatial arrays, the phase velocity can be estimated with several sensors simultaneously, rather than utilizing only two point estimates of phase change. The following sections introduce the common geotechnical phase velocity definitions and the traditional phase velocity estimation procedures.

2.4.1 Phase Velocity Definitions

Initial surface wave analysis methods yielded a *composite* dispersion curve, made of *apparent* phase velocities. The most recent surface wave velocity estimator yields an *effective* phase velocity and an *effective dispersion surface* (Lai, 1998). Before embarking on a detailed analysis of phase velocity estimators, the meaning of the different terms should be settled.

Figure 2.5 shows the unwrapped phase change as a function of distance for a multimodal wavefield. The asterisk points show the definition of apparent phase velocity, which is the average slope of phase change versus distance between two spatial lags. The effective phase velocity is defined by the instantaneous phase change with distance, i.e. as the distance between the two spatial lags tends to zero (shown with triangles in Figure 2.5). As Figure 2.5 shows, the effective phase velocity is the instantaneous apparent phase velocity. The ambiguity of the definitions stems from the poor spectral characteristics of traditional phase velocity estimators, which has led to several terms to define a phase velocity consisting of an undefined mix of modes.

The purpose of velocity measurements in material characterization, from an engineering perspective, is to determine a property of the material. In the case of apparent and effective phase velocity estimates, the estimated velocity is not only a function of the material, but also a function of the location of the measurement. Lai (1998) recognized that the effective phase velocity was a local quantity, i.e. a function of the spatial position of the measurement. In contradistinction, the modal phase velocity, as introduced in Chapter 6, does not depend on the location of the measurement and actually represents a natural property of the layered profile. Instead of relying on the location of the measurement, the ability to estimate modal phase velocities from advanced signal processing methods depends on the resolution and leakage control of the estimator.

2.4.2 Traditional Two-Point Estimators

The traditional estimate of the dispersion curve averages the two-point phase change estimates, given by the following wavenumber equation

$$k_{apparent}(f) = \frac{\Delta\phi(f)}{\Delta d} = \frac{1}{N}\sum_{\ell=1}^{N} \frac{\Delta\phi_{\ell}(f)}{\Delta d_{\ell}} \tag{2.1}$$

yielding the following Rayleigh wave apparent phase velocity estimate

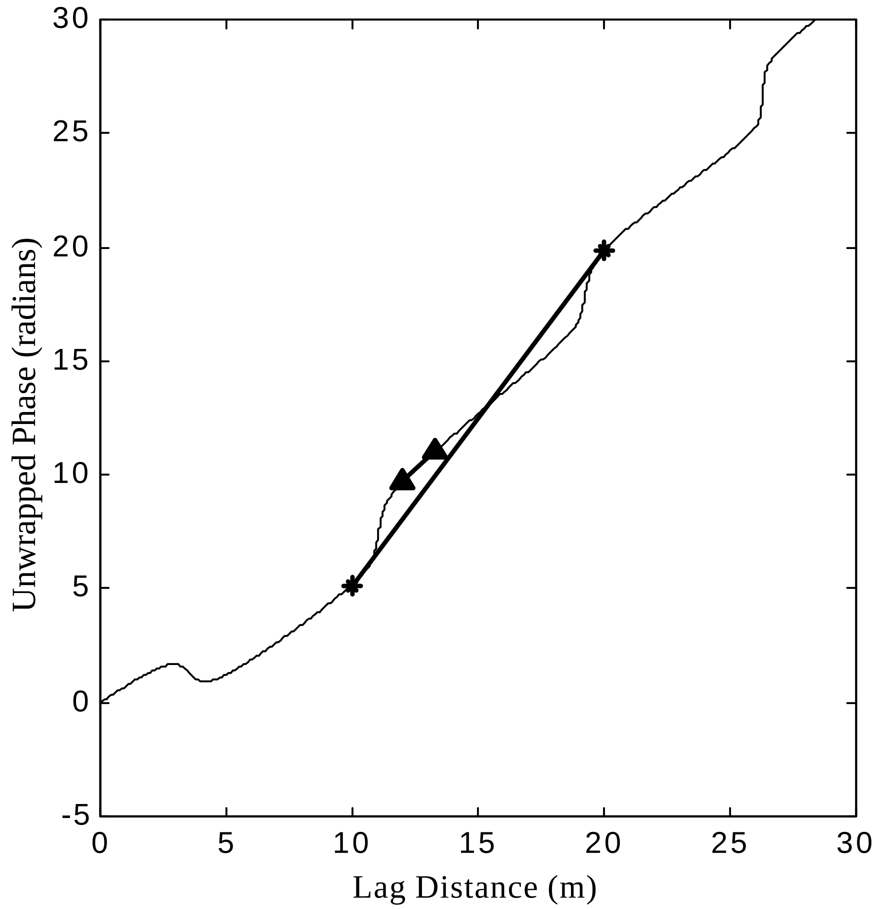

Figure 2.5 Apparent and Effective Phase Velocities. The apparent phase velocity is defined by the average slope between two points (shown with the asterisks), and the effective phase velocity is defined by the instantaneous slope between two points (shown with triangles).

$$V_R(f) = \frac{2\pi f}{k_{apparent}(f)} \tag{2.2}$$

where N = total number of spatial samples, $\Delta\phi_l$ = change in phase as a function of lag sample l and frequency f, Δd_l = the change in distance between sensors as a function of lag sample l, and V_R = the apparent Rayleigh phase velocity as a function of frequency. The two-point wavenumber estimation procedure shown in Equation 2.1 is essentially equivalent to using only two temporal samples to estimate frequency, and the dispersion curve estimated from Equation 2.2 is called the *composite* dispersion curve. The phase change for each lag is estimated using Bartlett's procedure (discussed in Chapter 3), typically averaging ten independent estimates of the cross power spectrum. Only the phase information is actually considered when estimating the dispersion curve.

The inclusion of the different spatial lags is determined through the coherence function and engineering judgement. The coherence between two measurements is given by

$$\gamma^2(f,\ell) = \frac{G_{12}(f,\ell)G_{21}^*(f,\ell)}{G_{11}(f,\ell)G_{22}(f,\ell)} \tag{2.3}$$

where $G_{12}(f,\ell)$ = the cross power spectrum between sensors 1 and 2 as a function of frequency and spatial lag, $G_{11}(f,\ell)$ = the autopower spectrum estimate of sensor 1, and * = complex conjugation. The coherence tells how the measured process at sensor 2 relates to the measured process at sensor 1. If the coherence equals one, the measurements at the

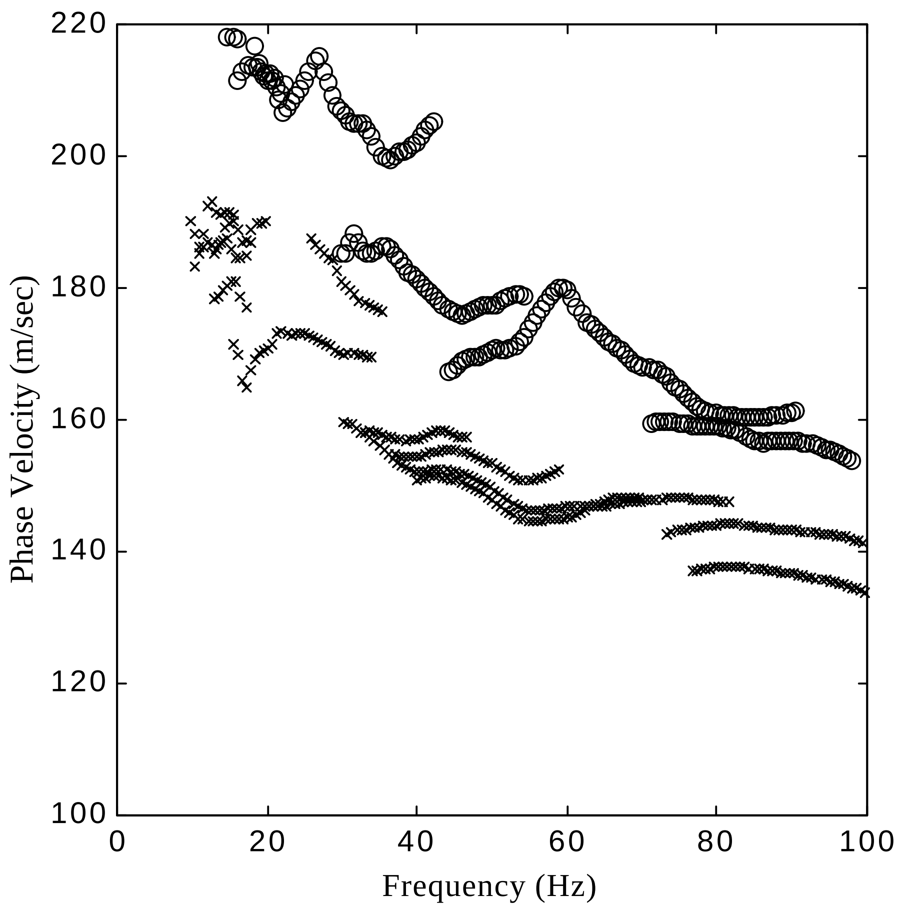

Figure 2.6 All Two-Point Phase Velocity Estimates for the ISC '98 Site. The traditional cross power spectrum method estimates are shown with circles, and the transfer function method estimates are shown with x's.

two sensors are linearly related, and the measurements at sensor 2 are considered to be due entirely to the motion measured at sensor 1. In typical analysis, only measurements with coherence greater than about 0.9 are included. Deterioration of coherence is due to the decrease in energy reaching the sensors at larger lags and the underlying seismic noise. The engineer may discard some spatial lags if they do not tend to fit the overall trend of the data, which introduces a subjective component into the analysis.

Figure 2.6 shows all the two-point phase velocity estimates for all spatial lags obtained at the 1998 International Site Characterization Conference site on Georgia Tech's campus, which will be referred to as the ISC '98 site. The estimates were obtained using the following traditional filtering criteria:

1.) Near-field: $d_1 > \lambda$,

2.) Receiver Spacing: $\Delta d \approx d_1$,

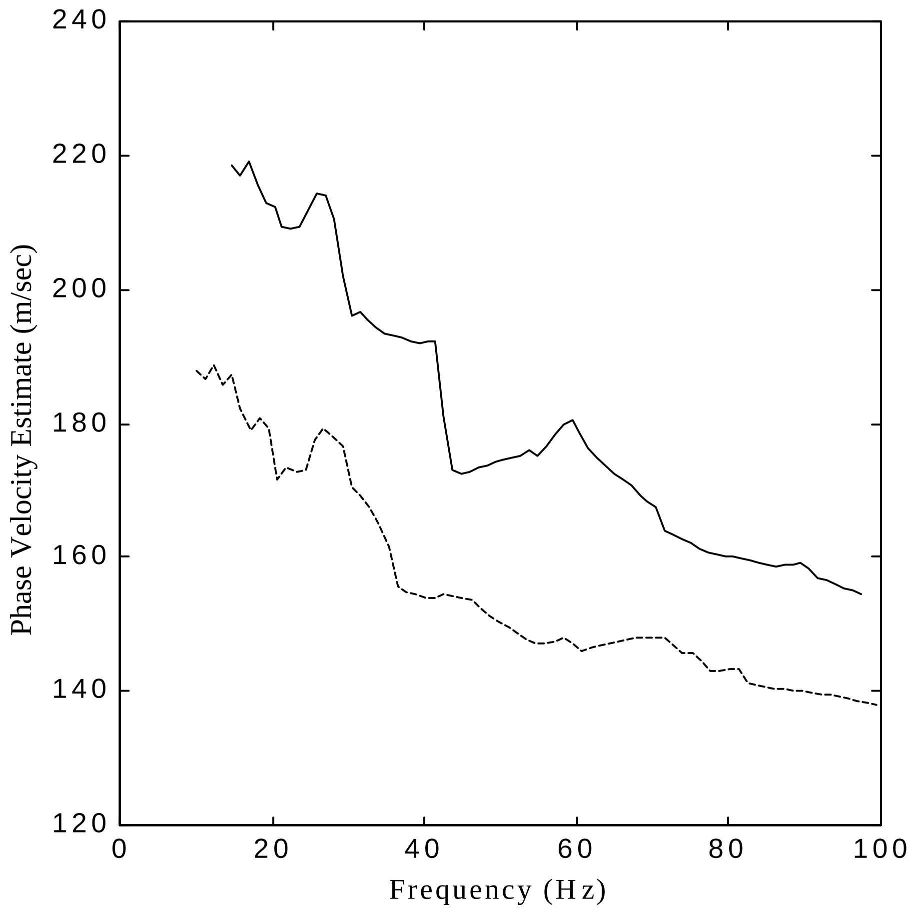

Figure 2.7 ISC '98 Site Composite Dispersion Curves from Traditional Two-Point Estimators. The composite dispersion curve from the traditional cross power spectrum method (solid line) and the transfer function method (dashed line) are shown.

where d_1 = the spacing between the source and the first receiver, and Δd = the spacing between the first and second receiver. The traditional filtering criteria are discussed completely in Chapter 6, especially in relation to the model incompatibility implications on the near-field.

The wide scatter of the data in Figure 2.6 is attributable to four factors. The difference between the theoretical ideal layered model and actual, variable model introduces variability in the phase velocity estimates. A model incompatibility between the theoretical model and the physics of cylindrically spreading waves affects the phase velocity estimation, as discussed in Chapter 6. The underlying random seismic noise introduces a variability into the estimates which may be quantified if the seismic noise field is stationary, and multiple propagation modes make the phase velocity a multiple valued function of frequency. The reason a definite trend appears in the difference between the traditional cross power and transfer function estimators is discussed in Chapter 6.

The composite dispersion curves for the data from Figure 2.6 are shown in Figure 2.7. The estimated dispersion curves show no ability to distinguish multiple modes. The

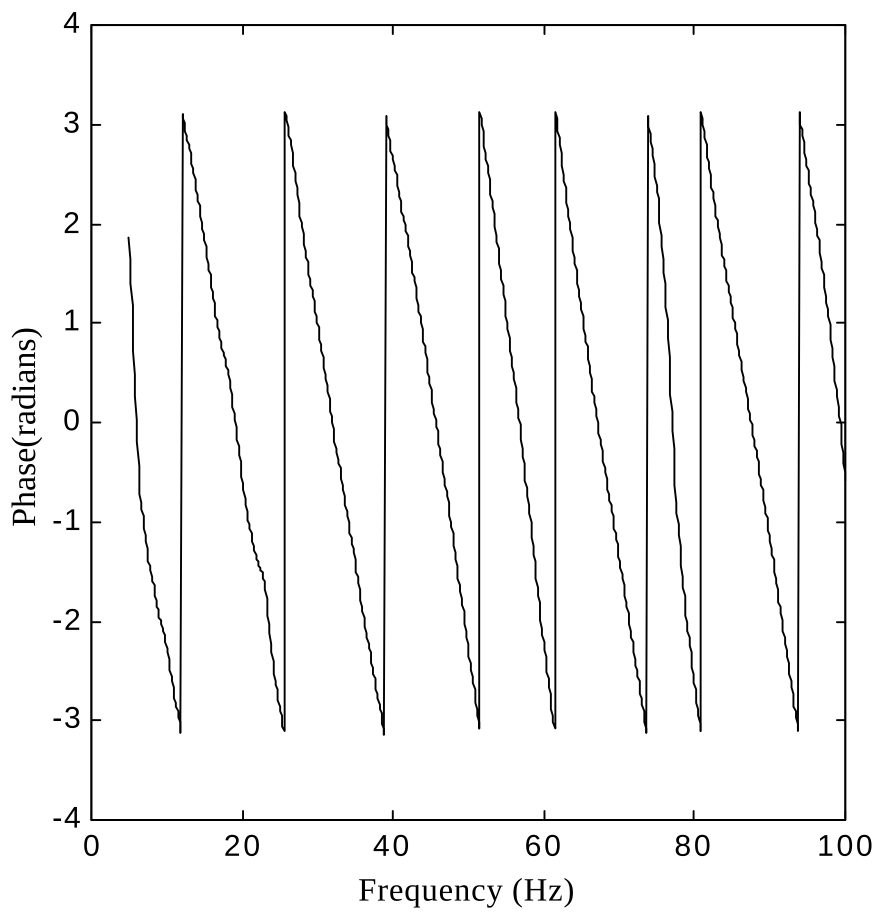

Figure 2.8 Phase Change as a Function of Frequency for a Spatial Lag = 10 m at the ISC '98 Site

phase change and coherence of the measurements as function of frequency for a single spatial lag equal to 10 m are shown in Figures 2.8 and 2.9. Also, Figure 2.10 shows a contour map of the coherence as a function of frequency and spatial offset. The plot follows expectations of coherence, and several salient points should be emphasized. Notice the decreased coherence at large spatial offsets for all frequencies, which decreases the ability to estimate a phase velocity at larger offsets and larger wavelengths. The coherence also tends to be lower at relatively high and low frequencies, which is primarily due to the limited energy output of the active harmonic source.

2.4.3 Least Squares Fitting of Wavenumber

The dispersion curve also has been estimated by least squares fitting a single wavenumber to spatial phase change data for each frequency (Mathews, et al., 1996; Lai, 1998). Only a single phase velocity can be estimated, and therefore, the least squares method suffers from the same drawbacks as the traditional analysis methods.

The least squares method displays no defense against errors in the measurements, no ability to handle multiple modes, and no recognition of the underlying random process. The

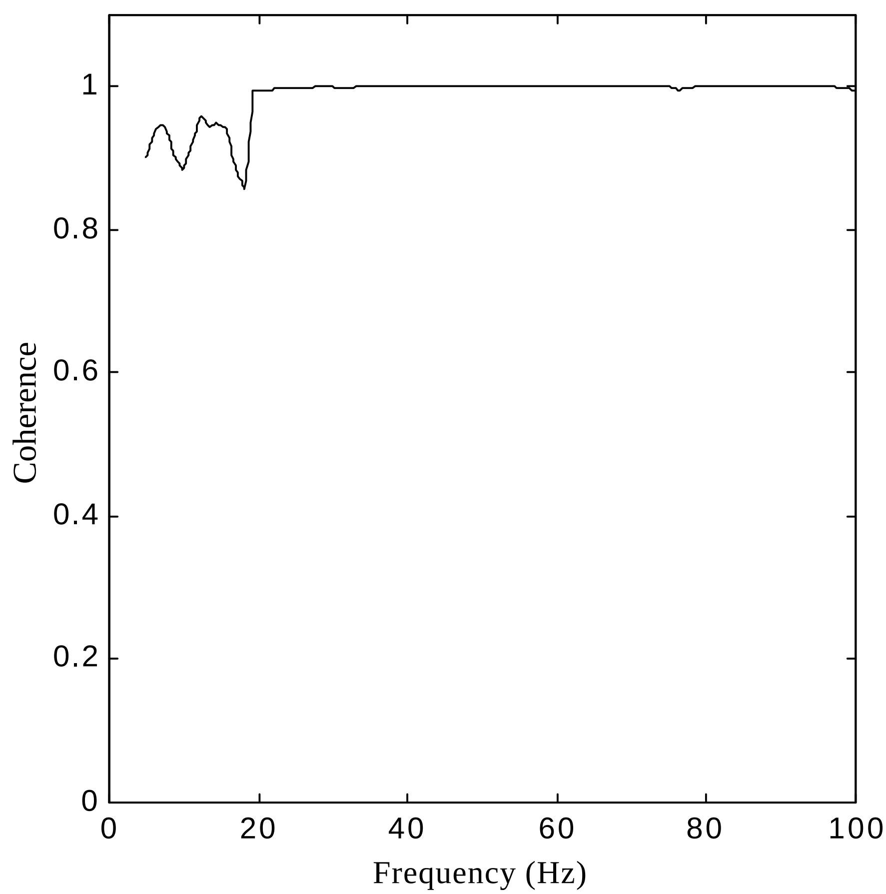

Figure 2.9 Coherence as a Function of Frequency for Spatial Lag = 10 m at the ISC ’98 Site

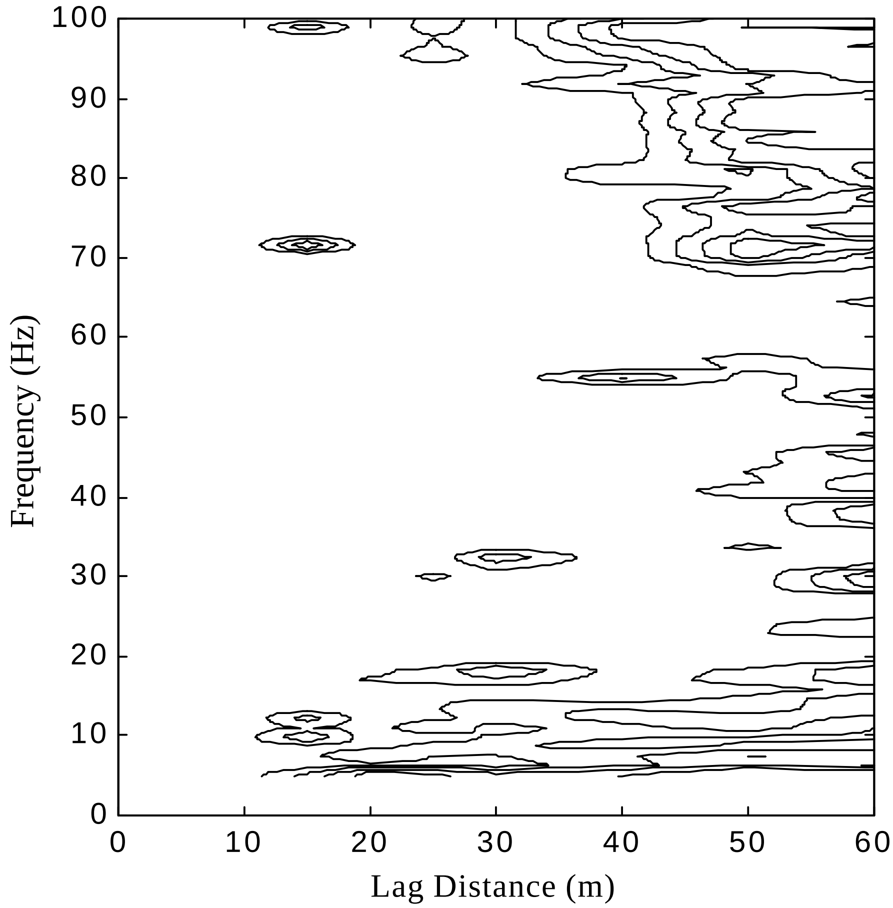

Figure 2.10 Contour Map of Coherence for ISC '98 Site. A larger density of lines indicates lower coherence.

results of least squares fitting a wavenumber differ depending on the type of source used. An impulsive source would decrease noise in the measurements due to stacking in the time domain, but the method is typically applied with a harmonic generator. Any noise left in the phase measurements may significantly affect the final estimate of wavenumber. The fitting of a least squares line is also affected by choice of receiver reference or zero phase reference, and has no ability to account for a single, sharp discontinuity in the phase data.

2.4.4 Multichannel Wavefield Transformation Method

Park et al. (1999) have implemented a spatial array method that is equivalent to a spatial Fourier transform. The method determines phase change as a function of distance, normalizes the magnitude to one to remove the effects of geometric spreading, and then uses trial wavenumbers to find peaks in an integral transformation.

Park et al. explain that equal weighting of the sensors, by normalizing magnitude, is used to account for the effects of attenuation and geometric spreading. Although the equal weighting does achieve the desired results, the uniform weighting may not be ideal for wavenumber estimation. The optimum beamforming and power estimation techniques discussed in Chapter 4 have distinct advantages over the wavefield transformation method. The resolution discussion in Chapter 6 will show the effects of normalizing a signal to unit magnitude and offer optimum normalization techniques.

2.4.5 Instantaneous Wavenumber

A suggested method to account for the presence of multiple modes calculates the effective Rayleigh phase velocity, or instantaneous velocity, with the instantaneous change in phase with distance. In the analysis, the phase is assumed to be continuous and smooth as a function of distance, i.e. the derivative exists at all points. If the phase change is continuous and smooth, the method would allow multiple modes of propagation to be considered in the inversion algorithm (Lai, 1998). In actual multiple mode propagation, the phase change data is not guaranteed to be smooth, as shown in Section 2.7, and the method would require samples spaced very close together to estimate an instantaneous change in phase versus distance. The effective phase velocity bears much resemblance to the instantaneous frequency concept in temporal domain signal processing. See McClellan, Schafer, and Yoder (1997) for a discussion of instantaneous frequency.

2.4.6 Frequency-Wavenumber Spectrum Analysis of Active Sources

Six sensor, linear arrays measuring an active surface wave harmonic source have yielded excellent experimental results compared to synthetic and theoretical expectations. The formulation of the problem using six sensors simultaneously allows a single mode to be extracted and the near-field interference to be reduced (Tokimatsu, 1995). The benefits in the spectral domain from using greater than two sensors will be discussed in Chapter 4.

2.4.7 Frequency-Wavenumber Spectrum Analysis of Passive Sources

In geotechnical engineering, Tokimatsu (1995) introduced the use of two-dimensional spatial arrays to estimate the dispersion curve. The method has worked well in practice, but the theory behind spatial arrays increases the robustness and understanding of the results. The detailed analysis of spatial array processing and theory will be covered in Chapter 4.

Briefly, the methods sample a seismic wavefield, assumed stationary, over various vector spatial lags. The phase velocity is estimated as a function of frequency and wavenumber by estimating a multidimensional power spectral density. If the wavefield has been sampled sufficiently, multiple sources and multiple modes may be isolated. In practice, this puts great restraints on the assumed wavefield and requires a large number of sensors.

2.5 Traditional Attenuation Curve Estimation

Material attenuation estimation is a more recent application of surface wave analysis in the geotechnical field. Since different wavelength surface waves sample the soil profile to varying depths, the attenuation of the surface waves varies as a function of frequency, depending on the material damping ratio of the soil layers. The most successful experimental estimates of material attenuation were made using the transfer function test setup (Lai, 1998) due to an established reference point. Passive wave attenuation measurements, introduced in Chapter 8, have not previously been made. This section discusses the traditional active attenuation estimation methods and their limitations.

2.5.1 Seismic Surface Wave Energy Dissipation

A general introduction to seismic surface wave energy dissipation is necessary. Energy dissipation can be categorized into the following three mechanisms:

1.) Geometric spreading,
2.) Apparent attenuation,
3.) Material attenuation.

Geometric spreading refers to the spreading of a fixed amount of energy over a larger area during propagation away from a source, e.g. cylindrically and spherically spreading waves. Apparent attenuation includes scattering, reflection, and mode conversion energy losses due to obstacles and material boundaries. Finally, material attenuation is an intrinsic property of the material being tested (Spang, 1995).

All references to Rayleigh wave attenuation estimates in this dissertation refer to material attenuation, and geometric spreading or apparent attenuation will explicitly be stated if under discussion. Geometric spreading, a major impediment in active surface wave attenuation measurements, will be discussed thoroughly in Chapters 6 and 7. To obtain correct material attenuation estimates, geometric spreading must be taken into account during the analysis.

2.5.2 Traditional Attenuation Estimation Model

Particle displacement magnitudes are experimentally measured as a function of distance from the source. Traditional attenuation coefficients are fit to the experimental data using the following general wavefield model:

$$A(\omega, r) = A_0(\omega)\, G\, e^{-\alpha(\omega) r} \tag{2.4}$$

where $A(\omega,r)$ = the magnitude of the experimentally measured particle displacement as a function of frequency and distance r from the active point source, $A_0(\omega)$ = the magnitude of the source or Rayleigh surface wave magnitude as a function of frequency, G = a function accounting for geometric spreading, discussed in Section 2.5.4, and $\alpha(\omega)$ = attenuation as a function of frequency.

Since the material attenuation enters into the model as an exponential power, changing to the natural logarithm domain allows the attenuation coefficient to become linear with spatial offset. Taking the natural logarithm of both sides of Equation 2.4 yields

$$\mathrm{Ln}\left(A(\omega, r)\right) = \mathrm{Ln}\left(A_0(\omega)\right) + \mathrm{Ln}(G) - \alpha(\omega) r \tag{2.5}$$

The experimental surface wave measurements at various spatial offsets supply $A(\omega,r)$ and r. Therefore, the parameters sought by the model are $\alpha(\omega)$, G, and $A_0(\omega)$. In some cases, the source magnitude is used for $A_0(\omega)$, which is incorrect, since only a portion of the source energy enters into Rayleigh surface wave modes. The following sections discuss the traditional methods of accounting for background noise, determining the regression intercept A_0, and accounting for geometric spreading of energy from active surface wave sources.

2.5.3 Traditional Noise Removal Techniques

Two traditional noise removal techniques have been used. Both methods work in a stationary noise environment, i.e. the background seismic noise statistics do not change during the measurement. Spang (1995) took an independent measurement of the noise field $A_{noise}(\omega)$ without the source present and subtracted the magnitude from the spectral magnitude measurements of the $A_{signal}(\omega) + A_{noise}(\omega)$. If the noise power changes during the measurement, the noise correction is incorrect.

The second noise removal technique uses the ordinary coherence function to correct the spectral amplitudes (Lai, 1998). A sensor placed on the source serves as the reference position to determine the coherence between the source and the receiver. The corrected amplitude, as a function of frequency and sensor position r, equals

$$\left|A_{Signal}(\omega, r)\right| = \gamma_{sr}(\omega, r)\left|A_{Signal+Noise}(\omega, r)\right| \tag{2.6}$$

where $A(\omega,r)$ = the spectral amplitude measured at spatial lag r and frequency ω, and γ_{sr} = the square root of the ordinary coherence function between the source and receiver, given by Equation 2.3 with the source and receiver replacing the indices 1 and 2. In traditional attenuation analysis, the measured cross spectrum is averaged several times before correcting with an average coherence. The averaging operation is linear only in stationary noise environments.

2.5.4 Traditional Geometric Spreading Models

Geometric spreading of energy from a point source complicates active surface wave material attenuation estimates. The problem has been compounded by the inability to estimate and account for multiple mode superposition effects.

2.5.4.1 Far-field Approximation

In the far-field, point source surface waves decay at a rate proportional to $\sqrt{r}$. The use of the $\sqrt{r}$ decay rate yielded the first attenuation estimates from active surface wave sources (Spang, 1995). The far-field model is

$$A(\omega, r) = \frac{A_0(\omega)}{r^{0.5}} e^{-\alpha(\omega) r} e^{j(\omega t - kr)} \tag{2.7}$$

where $A(\omega,r)$ = the spectral displacement magnitude measured at spatial offset r and frequency ω, and in this case, $G = r^{-0.5}$ is constant for all frequencies. Two parameters are necessary to fit the model - the attenuation coefficient $\alpha(\omega)$ and the initial signal amplitude $A_0(\omega)$.

Although the approximation has yielded seemingly acceptable attenuation coefficient estimates, the method 1.) Does not represent a spreading law, since the Rayleigh waves only "approach" a $\sqrt{r}$ decay with distance, 2.) Forces the experimental measurements to fit the incorrect physical model, and 3.) Fails to account for multiple modes. Chapter 7 fully discusses the inadequacies of the $\sqrt{r}$ model to correctly estimate active surface wave geometric spreading and material attenuation.

2.5.4.2 Geometric Spreading Function G(ω,r)

Numerical models of multiple mode wavefields indicated that the $\sqrt{r}$ decay model was inadequate to describe geometric spreading in different vertically heterogeneous profiles. To attempt to remedy this observation, a Rayleigh geometrical spreading function was introduced (Lai, 1998), leading to the following wavefield model:

$$A(\omega,r) = A_0(\omega)G(\omega,r)e^{-\alpha(\omega)r}e^{j(\omega t-\Psi(\omega,r))} \qquad (2.8)$$

where $A_0(\omega,r)$ = summation of all modal Rayleigh wave magnitudes for a given frequency, $G(\omega,r)$ = the geometrical spreading function, which is now a function of frequency and spatial offset, and $\psi(r,\omega)$ = the integrated phase argument from the source position to r due to the superposition of all modes of propagation. The geometrical spreading function and $\psi(\omega,r)$ are functions of both distance and frequency because they include the contribution of all the Rayleigh modes.

The geometrical spreading function is determined through an iterative and computationally expensive procedure, using the theoretical modes of propagation from a estimated soil profile to determine the theoretical spreading function for an active point source (Lai, 1998). The iterative procedure introduces additional uncertainty into the damping profile inversion process. Chapter 7 introduces the complete spectral representation of geometric spreading as $G(\omega,\mathbf{k})$.

2.5.5 Regression Model Intercept Estimation

In all of the attenuation estimation procedures, a Rayleigh wave amplitude must be determined. Two traditional methods have been used to determine the Rayleigh wave magnitude. An important distinction between wave magnitude and source magnitude should be emphasized. The source magnitude is the force input into the system by the harmonic or impulsive source; on the other hand, the Rayleigh wave magnitude only consists of the portion of the source energy entering into Rayleigh wave modes.

2.5.5.1 Regression Intercept

When using regression techniques, the Rayleigh wave magnitude may be estimated directly from the y-intercept. Allowing the intercept to be chosen through the regression optimization procedure allows the fitting of theoretical data to focus on matching the more distant spatial offset experimental measurements. As discussed in Chapter 7, the wave magnitude estimate from the regression intercept is incorrect when using the far-field geometric spreading model.

2.5.5.2 Active Source Experimentally Determined Magnitude

The active source magnitude, which can be determined through direct experimental measurements, has been suggested as the correct value to fix the intercept in the regression (Lai, 1998). As discussed in Chapter 7, the method is incorrect because only a portion of the source energy enters into the Rayleigh wave modes.

2.5.6 Traditional Attenuation Estimation Methods

Two traditional attenuation estimation methods have been implemented. The linear regression technique relies on a simple model design, and the non-linear method attempts to more accurately model superposition of modes and geometric spreading.

2.5.6.1 Traditional Linear Regression

The traditional linear regression estimator uses the $\sqrt{r}$ decay approximation and estimates a single attenuation coefficient as a function of frequency $\alpha(\omega)$. Noise is removed by measuring a reference noise magnitude before introducing the signal (Spang, 1995). The attenuation estimates suffer from the effects of the model incompatibility, i.e. incorrect source magnitude estimate and incorrect energy spreading, the inability to distinguish multiple modes, and the inability to account for changing noise magnitudes and statistics.

2.5.6.2 Non-Linear Regression

The traditional non-linear regression technique uses the theoretical multiple mode geometric spreading function $G(\omega,r)$, obtained using the method of reflection and

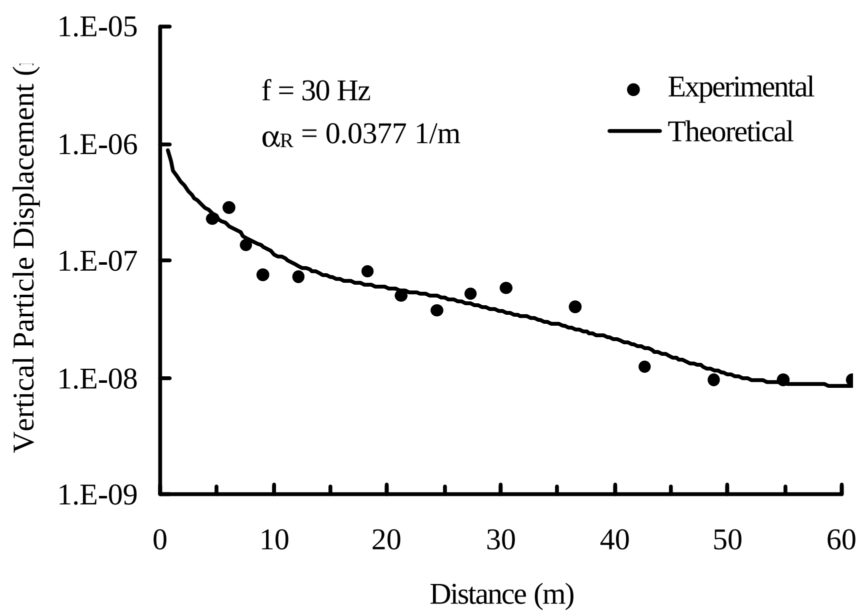

Figure 2.11 Traditional Non-Linear Material Attenuation Coefficient Estimate (From Lai, 1998)

transmission coefficients (Kennett, 1974), and fits a single multimodal attenuation coefficient as a function of frequency $\alpha(\omega)$ based on a non-linear fit of the theoretical multiple mode attenuation versus the experimental measurements (Lai, 1998). The solution recognizes the limitation of the $\sqrt{r}$ decay rate to match numerical models and experimental measurements. An example of a non-linear regression attenuation estimate is shown in Figure 2.11. Chapter 7 will introduce procedures to estimate modal attenuation coefficients from the linear model of Equation 2.5.

2.6 Inversion Procedure

An inversion procedure allows the estimation of the engineering properties of the near-surface earth from the experimentally measured dispersion and attenuation curves. The shear wave velocity profile is estimated from the dispersion curve, and the damping profile is estimated from the attenuation curve. Any errors or bias in the experimentally measured curves will propagate through the inversion procedure and introduce errors into the estimated soil profile. Experimental measurement of attenuation and dispersion are the primary focus of this dissertation. The reader is referenced to Lai (1998) and Spang (1995) for material relating to the inversion problem and algorithms. Figure 2.12 shows an example of an experimentally estimated dispersion curve and the inverted shear wave velocity profile.

The most commonly employed algorithms make a fundamental mode inversion. Fundamental mode inversions assume the experimental dispersion curve represents the fundamental mode of propagation, but as discussed previously, multiple modes exist in layered and gradually vertical heterogeneous profiles. In normally dispersive media, i.e. the shear wave velocity increases with depth, the fundamental mode has traditionally been

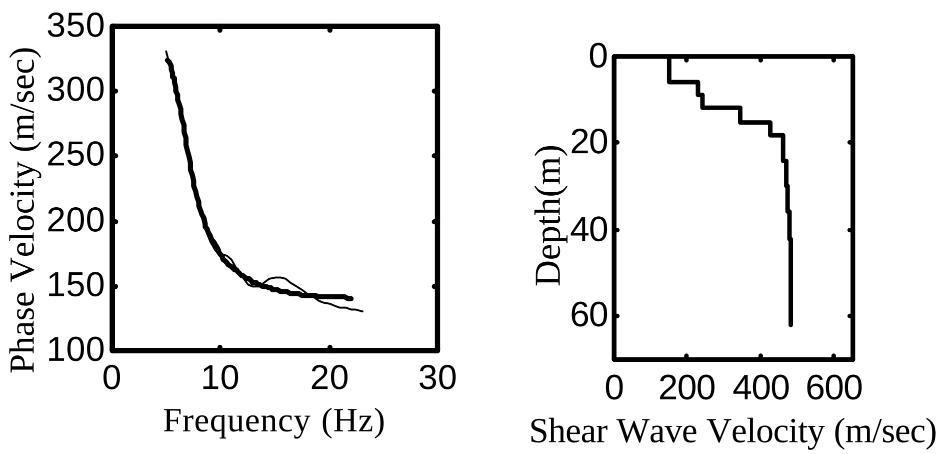

Figure 2.12 Example Dispersion Curve and Inverted Shear Wave Velocity Profile (From Zywicki and Rix, 1999). The left panel shows the experimentally estimated (light line) and the theoretical (dark line) dispersion curves. The inverted shear wave velocity profile is shown in the right panel.

assumed dominant, and a fundamental mode inversion has yielded apparently acceptable success. The presence of multiple modes can be modeled numerically and included in the inversion algorithm, but the procedure is computationally costly.

Traditionally, attenuation and phase velocity inversions were considered separately. Lai (1998) introduced a coupled inversion process to invert the experimentally measured curves simultaneously. The solution explicitly considers the coupled nature of damping and stiffness in the form of a complex-valued material parameter.

2.7 Summary

Geotechnical engineering tests relying on the analysis of seismic surface waves offer several advantages over lab testing, including the noninvasive nature of the tests. The test procedures and analysis have advanced considerably since the introduction of the Steady State Vibration Test, but the traditional analysis of phase velocity and attenuation continue to display noteworthy limitations. Two of the most important limitations are the following: 1.) The traditional two-point estimators lead to ambiguous phase velocity definitions, ad hoc testing recommendations, and an inability to extract single modes, and 2.) The model incompatibility, i.e. modeling cylindrically spreading surface waves with a plane wave model, has led to an increasingly complex surface wave phase velocity and attenuation model. The introduction of advanced signal processing methods and the correct cylindrical wavefield model will increase the abilities of geotechnical seismic surface wave tests. The following sections review the limitations of traditional seismic surface wave analysis procedures.

2.7.1 Summary of Traditional Dispersion Curve Estimators

The traditional phase velocity estimators fail to overcome all the identified difficulties. The failure is due to several reasons, and different methods achieve different levels of success. The following sections identify the primary impediments to obtaining optimum engineering phase velocity estimates from Rayleigh surface waves. Chapters 6 and 8 will introduce additional signal processing and modeling methods to overcome some of the listed impediments.

2.7.1.1 Inadequacy of Traditional Two-point Estimators

The traditional two-point dispersion curve estimators rely on a poorly defined problem and strong statistical expectations. The procedure of estimating spatial frequency content from two spatial offset samples is equivalent to attempting to estimate temporal frequency from only two temporal samples. Rather than explicitly optimizing an objective function based on the physics of wave propagation, the traditional method averages several two-point phase change estimates. The procedure implicitly assumes phase velocity estimates represent an underlying Gaussian process. By averaging several two-point phase velocity estimates, the estimate would asymptotically approach the expected value of the phase velocity and the variance would decrease with more samples. Over the spatial lags typically encountered, phase velocity estimates do not approach a Gaussian distribution due to the presence of multiple modes. The theory of random processes and advanced signal processing will allow the two-sensor problem to be reformulated as a synthetic linear array problem, and optimum phase velocity estimates to be obtained from a source and two sensors.

2.7.1.2 Reliance on Phase Data Only

Several of the traditional estimators rely only on the phase change data with distance or frequency. The magnitude of particle motions holds a great deal of information, especially pertaining to multiple modes. The traditional two-point estimator and the least squares fitting method allow no ability to include magnitude information in the phase velocity estimation procedure. Chapter 6 will discuss the disadvantages of ignoring magnitude information and offer alternatives to optimally estimate multiple modes.

2.7.1.3 Near-field Effects

The traditional sensor setup recommendations focus on mitigation of near-field effects. The original study comparing numerical models to study the effects of sensor placement (Sanchez-Salinero, 1987) relied on a model incompatibility. Unfortunately, the model incompatibility between actual cylindrically spreading surface waves and plane wave propagation has been used in SASW analysis since its introduction into the geotechnical literature. The numerical or theoretical models properly account for body waves and cylindrical spreading of energy, but the phase velocity is estimated by fitting a plane wave model to the experimental data. Part of the near-field effects actually represent the incompatibility of model choice, and Chapter 6 will explicitly analyze the effects of poor model choice, suggest alternatives to improve surface wave modeling, and analyze the causes of the results traditionally attributed to the near-field.

2.7.1.4 Multiple Modes and Traditional Modal Superposition Assumptions

In multiple mode propagation, the phase change with distance may be discontinuous, as shown in Figure 2.13, where plane wave propagation has been assumed. The figure shows a wavefield containing two modes propagating with equal amplitude. The light lines represent the individual modal phase changes, the medium line represents the superposition of the two modes, and the dark line shows the normalized equivalent wavenumber, i.e. each wavenumber is weighted by its amplitude and summed to give an equivalent wavenumber.

In this example, the derivative does not exist at several spatial lags due to jumps in the phase change data. When the derivative does exist, it may not actually correspond to any of the modal velocities. In layered soil profiles, the phase change data probably will be smoothed due to the presence of more than two modes in varying degrees. If the data in Figure 2.13 were collected experimentally, the phase velocity estimate would actually converge to the second modal velocity. If data were only available to a distance of about 4 m, the phase velocity would correspond to the equivalent wavenumber, and in some spatial locations near the phase jumps, the velocity estimate would be negative Therefore, the estimated phase velocity may or may not correspond to a single mode or average of all the modes present. If a least squares wavenumber was fit to the superposed phase data in Figure 2.13, the estimated phase velocity would depend considerably on the spatial locations of the samples.

In multiple mode wavefield propagation, the estimated phase velocity has traditionally been assumed to be primarily the fundamental mode or a weighted average of the modes present. As shown in Figure 2.13, the estimated phase velocity may represent a single mode, even when other modes are present in equal amounts. The traditional SASW methods offer no ability to guess which mode or mix of modes is contained in the phase velocity estimate. The ambiguity of the estimate stems from consideration of phase change data only and simple two-point analysis methods.

Figure 2.14 shows an additional graph of phase change information for a wavefield containing five modes. The wavelengths of the different modes vary from 5 to 36 m. A few more salient features are important to notice in Figure 2.14. The superposed phase varies from a linear trend considerably over the spatial distance of 30 m. The superposed phase also does not match a single mode or the equivalent wavenumber. The common assumption of how different modes are interacting to yield a composite dispersion curve in the traditional analysis needs additional consideration.

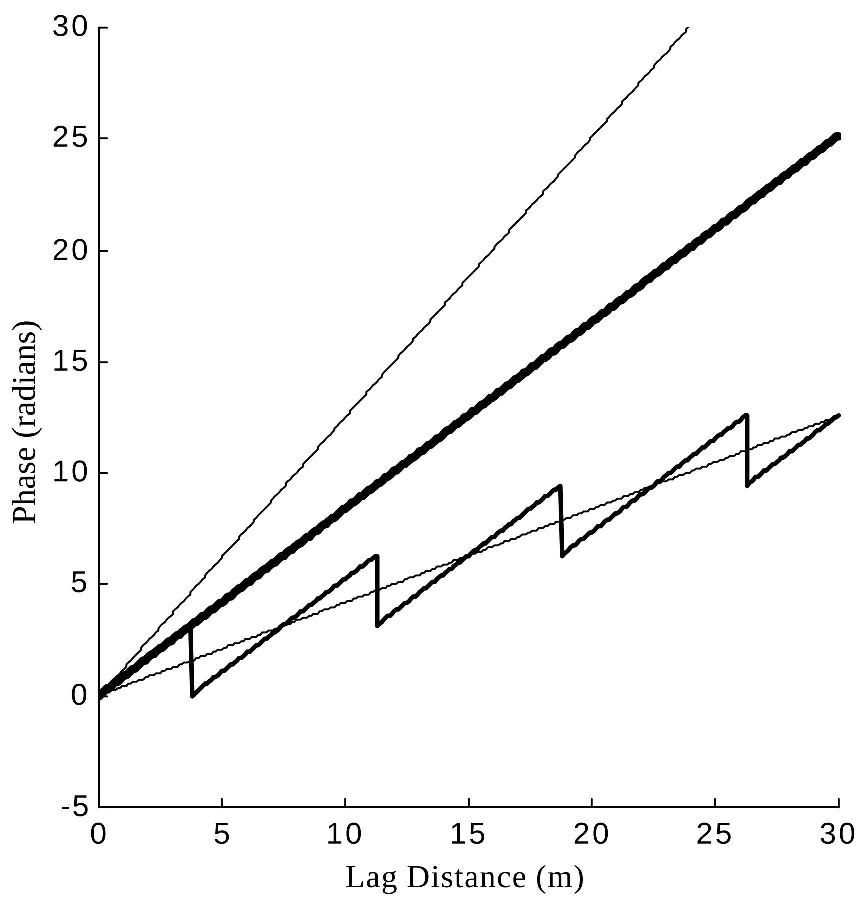

Figure 2.13 Phase Change Versus Distance for a Wavefield Containing Multiple Modes (light lines = modal phase change, medium line = superposed phase change, dark line = equivalent phase change, calculated as sum of weighted wavenumbers).

2.7.1.5 Single Reference Point

The traditional methods that estimate phase velocity from a single reference point, e.g. traditional two-point transfer function method and least squares wavenumber fit, all suffer from the same uncertainties. A sharp, lateral discontinuity in the soil profile or smooth spatial variability in the soil properties will cause estimation errors. Advanced signal processing techniques offer methods that do not rely on a single reference point.

2.7.1.6 Qualitative and Subjective Analysis Procedures

The traditional analysis procedures require engineering judgement regarding inclusion of data and depend on poorly defined and qualitative filtering criteria. The filtering criteria have typically been developed from visual analysis of numerical and experimental data, rather than relying on fundamental signal processing and wave propagation concepts. The inability to clearly define filtering criteria stems from the model incompatibility and simplistic signal processing methods.

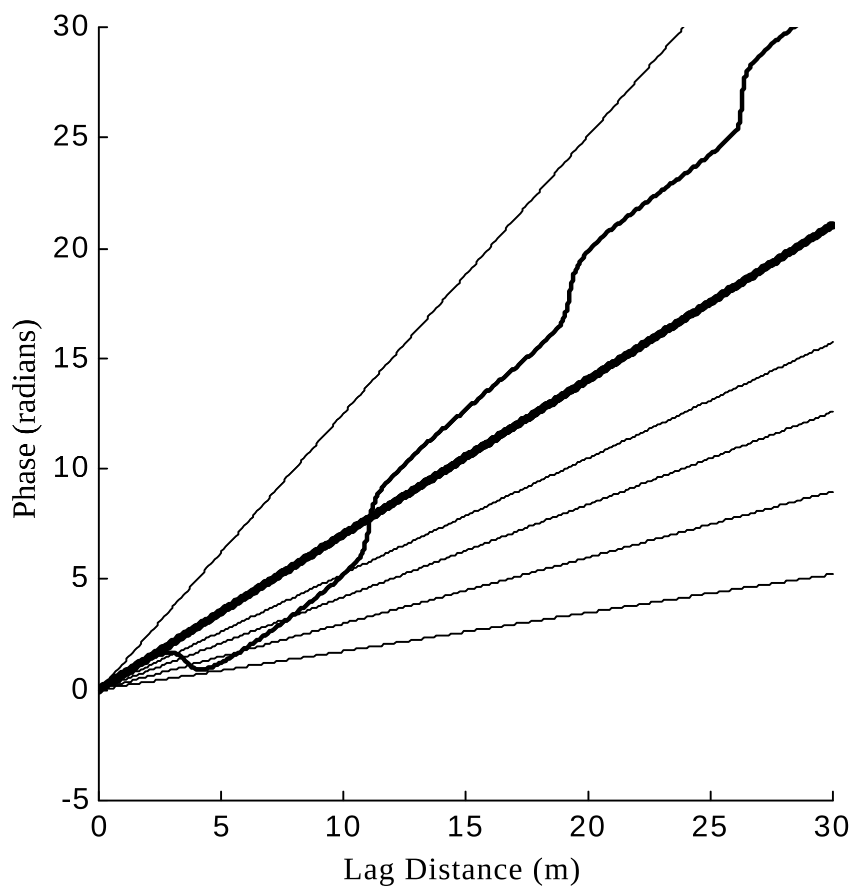

Figure 2.14 Phase Change Versus Distance for a Wavefield Containing Five Modes (light lines = modal phase change, medium line = superposed phase change, dark line = equivalent phase change, calculated as sum of weighted wavenumbers).

2.7.2 Summary of Traditional Attenuation Estimators

Attenuation measurements are a relatively recent application of surface waves, and the transfer function method represents a significant improvement in experimental data collection. Advanced signal processing algorithms and an understanding of random processes offer significant improvements in the analysis of attenuation of surface waves. The following sections identify some of the areas requiring additional consideration. Chapter 7 will explore methods to handle the active source problems, and Chapter 8 will discuss attenuation estimates from passive wave measurements.

2.7.2.1 Model Incompatibility

The wave propagation model incompatibility discussed previously introduces an error into the attenuation estimate because the magnitude is assumed to come from a plane wave propagation field with an artificially introduced geometric spreading function, when in fact, the magnitude comes from a cylindrically spreading wavefield. The experimental measurements are forced to fit an incorrect physical model, affecting the attenuation estimates. Chapter 7 will explore the effects of the model incompatibility on attenuation measurements and introduce the correct Hankel function cylindrical wave equation solution.

2.7.2.2 Geometric Spreading

Geometric spreading represents one of the most significant impediments to obtaining optimum attenuation measurements from active surface waves. Lai (1998) introduced a method that calculates a theoretical geometric spreading function prior to the inversion for the damping ratio profile. The model also lumps all the modes into a single attenuation estimate, while in reality, different Rayleigh modes will propagate with different material attenuation and geometric spreading rates. Chapter 7 introduces new analyses techniques that completely account for the geometric spreading of energy in active surface wave tests. The new methods use the correct Hankel function solution and a complete temporal and spatial spectral wavefield model, in opposition to the lumping of several modes into one geometric spreading function. Passive wave measurements offer a promising alternative to estimate attenuation coefficients. Since most passive wave measurements are in the far-field, the plane wave assumption may be invoked, and therefore, geometric spreading is no longer a factor.

2.7.2.3 Multiple Modes

Traditional attenuation methods do not experimentally account for multiple modes of propagation. Chapter 7 will discuss the effects of modal superposition on active surface wave energy dissipation. The superposition of multiple modes changes the rate of geometric spreading and controls oscillations around the longest wavelength mode.

2.7.2.4 Seismic Background Noise

A significant impediment to attenuation estimation is the presence of nonstationary seismic noise. When using a harmonic source, the variable magnitude of the background noise introduces a variable additive magnitude with spatial lag. Poor noise removal techniques affect the attenuation coefficient estimate.