A Fast Calculation of Frequency-Wavenumber Spectrums with the Fast Fourier Transform

A.1 Introduction

If the sensor locations can be placed on an underlying uniformly spaced grid, the FFT can be exploited for efficient implementation of power spectrum estimation. The implementation differs in the linear and two-dimensional array case, and the two-dimensional array requires an additional dimension of calculation. The use of the FFT is presented for non-uniform linear arrays.

A.2 Efficient Implementation of Linear Array Power Spectrum Estimators

If spatial samples are obtained along a non-uniformly spaced linear array, the samples may be placed on an underlying uniformly spaced linear grid. The uniform grid is made by placing zeros at positions where no sensor exists on the uniform grid. For example, if the first three sensor positions are 1, 2 and 5 m, and the array contains n sensors, then the spatiospectral correlation matrix equals

$$R(\omega) = \begin{bmatrix} S_1S_1^* & S_1S_2^* & 0 & 0 & S_1S_3^* & 0 & \mathrm{i} & S_1S_n^* \\ S_2S_1^* & S_2S_2^* & 0 & 0 & S_2S_3^* & 0 & \mathrm{i} & S_2S_n^* \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{i} & \mathrm{j} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{i} & \mathrm{j} \\ S_3S_1^* & S_3S_2^* & 0 & 0 & S_3S_3^* & \mathrm{i} & \mathrm{i} & S_3S_n^* \\ 0 & 0 & 0 & 0 & \mathrm{j} & \mathrm{l} & & \mathrm{j} \\ \mathrm{j} & \mathrm{j} & \mathrm{j} & \mathrm{j} & \mathrm{j} & & \mathrm{l} & \mathrm{j} \\ S_nS_1^* & S_nS_2^* & \mathrm{i} & \mathrm{i} & S_nS_3^* & \mathrm{i} & \mathrm{i} & S_nS_n^* \end{bmatrix} \qquad (A.1)$$

For the power spectrum estimators discussed in Chapter 4, the FFT is implemented by taking the FFT down the columns and then across the rows of either $R(\omega)$ or $R(\omega)^{-1}$. The power spectrum estimates along the linear axis of the array fall along the main diagonal of the resulting matrix, which has dimensions (Length of FFT) x (Length of FFT).

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