A New Method of Underground Structure Estimation Using Microtremors

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1 Introduction[1]

Microtremors are the background motions of the solid Earth which occur continuously and are apparently unrelated to specific event such as earthquakes and explosions. In seismology, the microtremors are taken as a seismic noise which makes it difficult to identify the specific events in seismic records.

The components of microtremors with dominant frequencies usually above 1 Hz are considered to be generated by various human activities such as trafic, industries, construction work etc. so called cultural noise. Atmospheric and oceanographic disturbances also cause the generation of microtremors of which the dominant frequencies occupy a broad low-frequency range from less than 0.01 Hz to, say, 0.5 Hz, i.e. period from 1 s to more than 100 s.

Seismograms recorded at permanent or temporary seismic stations frequently show that the amplitudes and periods of microtremors vary very often with high irregularity, and their variation depends on where and when the seismograms are recorded. The variations also show that the complete behavior of microtremors could not be described in terms of a mathematical formula which can express amplitudes of microtremors as a deterministic function of both time and space. Practically, we cannot theoretically determine an exact amplitude of microtremors at a time in a given place. Considering from the probability theory, we could say that the amplitude of microtremors at a time in a place is a random variable, so that the complete function of the amplitude against time should be called a random process.

If we observe microtremors at a place for a long time and divide the observed record into a number of time intervals, microtremors in those intervals seem much the same one another. This implies that the statistical properties of microtremors are unchangeable over the time observed, i.e. changing of microtremors in amplitude and period at a place is almost the same in all the time intervals; consequently, the microtremor can be called a stationary random process.

Considering from the standpoint of seismic wave theory, microtremors consist

[1]This note is revised in July, 1997.

of surface waves such as Rayleigh and Love waves, and body waves from many sources randomly distributed in azimuth and distance around the observation point. Some portions of the microtremors are nonpropagating and appear to be incoherent between records from different sensors separated by small spatial lags. Of these waves, the surface waves must mainly constitute the microtremors, since the greater part of energy of microtremors must be supplied with the vibration energy due to elastic disturbance on the surface of the Earth.

According to the fact that the microtremors must involve the surface waves, observations of microtremors should provide information about the underground structure at the observation site, because the surface waves have such a dispersion property that phase velocities are changed with frequency or period depending on the underground structure.

The problem of measuring phase velocities of surface waves in microtremors has been studied by a number of authors. In these studies, there are two principal methods to be applicable to the measurement of phase velocities: the frequency-wavenumber (f-k) method (Capon, 1969) and the spatial autocorrelation (SPAC) method (Aki, 1957).

The f-k method makes use of a seismic array consisting of a number of seismometers spatially placed. The method was originally developed in Montana, USA to discriminate between earthquakes and underground nuclear explosions using a large aperture seismic array (LASA) (e.g. Green et al., 1966). Actually, the algorithm of the method applied to the observation of microtremors in the period range shorter than 3 or 4 s has well provided undergroud structures down to depths of hundreds meters (Horike, 1985) and to depths of a few thousand meters (Matsushima and Okada, 1990), although the spatial extent of seismometer arrays they employed was extremely smaller than the LASA.

The SPAC method was originally developed by Aki (1957). This method, using a special array, estimates spatial autocorrelation coefficients of microtremors from which phase velocities of surface waves can be estimated. Observations to be employed in the SPAC method can be made with a relatively small number of seismometers (Okada and Sakajiri, 1983) and data processing and analyzing for the method are so simple that even personal computers are able to process. The method is helpful not only when reconnaissance surveys or quick results are required, but also when surveys have to be made within a limited budget. Furthermore, this method is able to estimate underground structures explicitly for S-wave velocities, because the method can identify Love waves in microtremors also in a dispersion form, if the observation is made by a circular array with 3-component seismometers.

2 Stationary Process

Let $X(t, \boldsymbol{\xi})$ be a stationary random (stochastic) process which depends on a time parameter t and a location vector in the plane $\boldsymbol{\xi}\,(x, y)$. The process will have a spectral representation (Yaglom, 1961; Priestley,1981)

$$X(t,\xi) = \int\!\!\int\!\!\int_{-\infty}^{\infty} \exp\left(i\omega t + i\boldsymbol{k}\,\boldsymbol{\xi}\right) dZ(\omega,\,\boldsymbol{k}) \tag{1}$$

where ω is the angular frequency (in radians per unit time), $\boldsymbol{k}$ the wavenumber (in radians per unit distance) and Z is a random spectral process. This means that any stationary random process in time and space can be regarded as a continuous sum of independent waves with different frequencies ω and wavenumber $\boldsymbol{k}$. In other wards, any stationary process can be represented as the sum of sine and cosine functions with random coefficients $dZ(\omega,\,\boldsymbol{k})$.

The stochastic process $Z(\omega,\,\boldsymbol{k})$ has the special property that its increments at different values of ω and $\boldsymbol{k}$ are uncorrelated, i.e. for any two distinct frequencies, ω, ω' and also for any two distinct wavenumbers $\boldsymbol{k}$, $\boldsymbol{k}'$, the random variables,

$$dZ(\omega,\,\boldsymbol{k}) = \{Z(\omega + d\omega,\,\boldsymbol{k} + d\boldsymbol{k}) - Z(\omega,\,\boldsymbol{k})\},$$

and

$$dZ(\omega',\,\boldsymbol{k}') = \{Z(\omega' + d\omega',\,\boldsymbol{k}' + d\boldsymbol{k}') - Z(\omega',\,\boldsymbol{k}')\}$$

are uncorrelated. The process $Z(\omega,\, \boldsymbol{k})$ is called a 'doubly orthogonal process.' In addition, there must obviously be some intimate relationship between the properties of the process $Z(\omega,\, \boldsymbol{k})$ and spectral properties of $X(t,\, \boldsymbol{\xi})$. It turns out that this relationship is most conveniently expressed in terms of the function $H(\omega,\, \boldsymbol{k})$, the (non-normalized) integrated spectrum of $X(t,\, \boldsymbol{\xi})$, that is,

$$E[|dZ(\omega,\, \boldsymbol{k})|^2] = dH(\omega,\, \boldsymbol{k}), \tag{2}$$

where E denotes the mean of expectation, i.e. at each frequency ω, and each wavenumber $\boldsymbol{k}$, the increment in $H(\omega,\, \boldsymbol{k})$ is equal to the mean of the squared amplitude of the corresponding component in eq.(1). When $X(t,\, \boldsymbol{\xi})$ has a purely continuous spectrum, the increment in $H(\omega,\, \boldsymbol{k})$ may be expressed as

$$dH(\omega,\, \boldsymbol{k}) = h(\omega,\, \boldsymbol{k})d\omega\, d\boldsymbol{k}$$

$h(\omega,\, \boldsymbol{k})$ being the (non-normalized) spectral density function, then eq.(2) reduces to

$$E[|dZ(\omega,\, \boldsymbol{k})|^2] = h(\omega,\, \boldsymbol{k})d\omega\, d\boldsymbol{k} \tag{3}$$

Thus the process $Z(\omega,\, \boldsymbol{k})$ has the following properties;

(i) $\quad E[dZ(\omega,\, \boldsymbol{k})] = 0,\quad$ for all ω and $\boldsymbol{k}$, $\quad$ (4)

(ii) $\quad E[|dZ(\omega,\, \boldsymbol{k})|^2] = dH(\omega,\, \boldsymbol{k}),\quad$ for all ω and $\boldsymbol{k}$, $\quad$ (5)

and (iii) for any two distinct frequencies $\omega,\, \omega'$ $(\omega \neq \omega')$ and two distinct wavenumbers $\boldsymbol{k}, \boldsymbol{k}'$ $(\boldsymbol{k} \neq \boldsymbol{k}')$

$$E\left[dZ^*(\omega,\, \boldsymbol{k})dZ(\omega',\, \boldsymbol{k}')\right] = 0 \tag{6}$$

where * denotes the complex conjugate.

In our case it will be convenient to consider the representation of the process in polar form. Let

$$\boldsymbol{\xi} = r(\cos\theta,\, \sin\theta), \quad \text{and} \quad \boldsymbol{k} = k(\cos\phi,\, \sin\phi)$$

so that a stationary random process $X(t, \boldsymbol{\xi})$ is

$$X(t, r, \theta) = \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{2\pi} \exp\{i\omega t + irk\cos(\theta - \phi)\} d\zeta(\omega, k, \phi) \tag{7}$$

where

$$\begin{aligned} dZ(\omega, \boldsymbol{k}) &= k dZ'(\omega, k, \phi) \\ &= d\zeta(\omega, k, \phi) \end{aligned}$$

This means that any stationary process is the sum of independent waves coming from direction ϕ with angular frequency ω and wavenumber k.

Since microtremors can be regarded mainly as an ensemble of the surface waves, we assume that at each angular frequency ω, the energy is concentrated at a single wavenumber; that is, the velocity is a single valued function of frequency. This means that only one mode of the surface waves could be extracted from microtremors. If we observe the vertical component of microtremors, the dispersive Rayleigh waves can be chosen as the subject to study. The above assumption corresponds to the spectral process ζ being concentrated on a curve $[\omega, k(\omega)]$, then eq.(7) may be reduced to

$$X(t, r, \theta) = \int_{-\infty}^{\infty} \int_0^{2\pi} \exp\{i\omega t + irk(\omega)\cos(\theta - \phi)\} d\zeta(\omega, \phi) \tag{8}$$

Here we assume that

$$\begin{aligned} \mathrm{E}[|d\zeta(\omega, \phi)|^2] &= dH(\omega, \phi) \\ &= h(\omega, \phi) d\omega\, d\phi \end{aligned} \tag{9}$$

where $H(\omega, \phi)$ is the frequency-direction integrated spectrum of X, and $h(\omega, \phi)$ is the frequency- direction spectral density which gives the average energy at frequency ω arriving from direction ϕ.

3 Spatial Autocorrelation Method

3.1 Vertically Polarized Waves in Microtremors

If microtremors can be regarded as an ensemble of the surface waves, the observation of the horizontal component may record both Rayleigh and Love waves, while the observation of vertical component can record the Rayleigh waves only.

In the present section, we discuss the vertically polarized waves in microtremors, which may be considered as an ensemble of the dispersive Rayleigh waves.

3.1.1 Autocovariance Function

We define an autocovariance function at a location (r, θ) as

$$\begin{aligned} R(\tau, r, \theta) &= E[X^*(t, r, \theta)\, X(t+\tau, r, \theta)] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{2\pi}\int_{0}^{2\pi} \exp[i(\omega' - \omega)t + i\omega'\tau + ir\{k'\cos(\theta - \phi') - k\cos(\theta - \phi)\}] \\ &\qquad \cdot E[d\zeta^*(\omega, \phi) d\zeta(\omega', \phi')], \end{aligned} \tag{10}$$

where X^* is the complex conjugate of X. Since $X(t, r, \theta)$ is a stationary process for all time t and space (r, θ), the left hand side of eq.(10) is the aoutocovariance function, and is therefore a function of τ only and does not depend on t and (r, θ). This can be so only if $E[d\zeta^*(\omega, \phi)\, d\zeta(\omega', \phi')] = 0$ for all $\omega \neq \omega'$ and $\phi \neq \phi'$. Hence, eq.(10) is reduced to

$$R(\tau) = \int_{-\infty}^{\infty}\int_{0}^{2\pi} \exp(i\omega\tau) E[|dzeta(\omega, \phi)|^2], \tag{11}$$

and, using eq.(9),

$$\begin{aligned} R(\tau) &= \int_{-\infty}^{\infty}\int_{0}^{2\pi} \exp(i\omega\tau) h(\omega, \phi) d\omega d\phi \\ &= \int_{-\infty}^{\infty} \exp(i\omega\tau) \left[\int_{0}^{2\pi} h(\omega, \phi) d\phi\right] d\omega. \end{aligned} \tag{12}$$

If we calculate the azimuthal sum of the frequency-direction spectral density $h(\omega, \phi)$, the spectrum of the process at a single point in space, $h_0(\omega)$, is obtained, that is,

$$h_0(\omega) = \int_0^{2\pi} h(\omega, \phi)\, d\phi, \tag{13}$$

then $R(\tau)$ can be written as

$$R(\tau) = \int_{-\infty}^{\infty} \exp(i\omega\tau) h_0(\omega) d\omega. \tag{14}$$

If $R(\tau)$ decays to zero 'fast enough' (as $\tau \to \pm\infty$), the spectrum of the process at a single point in space, $h_0(\omega)$, is given by the Fourier transform of the autocovariance function $R(\tau)$, that is,

$$h_0(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-i\omega\tau) R(\tau) d\tau. \tag{15}$$

Setting $\tau = 0$ in eq.(14) we have

$$R(0) = E[|X(t, r, \theta|^2] = \int_{-\infty}^{\infty} h_0(\omega) d\omega, \tag{16}$$

which represents the total power (i.e. the power contributed by all frequency components) of the process (Priestley, 1981).

3.1.2 Spatial Autocorrelation Coefficient

For the observation to derive phase velocities from microtremors we consider a special design of array, which consists of seismometers equally spaced on a circle and one seismometer at the center. The stationary random process at the center of the array may be written from eq.(8)

$$X(t, 0, 0) = \int_{-\infty}^{\infty} \exp(i\omega t) d\zeta(\omega, \phi), \tag{17}$$

and also the process at a point (r, θ) on a circle with radius r may be written

$$X(t, r, \theta) = \int_{-\infty}^{\infty} \int_0^{2\pi} \exp\{i\omega t + irk\cos(\theta - \phi)\} d\zeta(\omega, \phi). \tag{18}$$

For these processes, we define a spatial autocovariance function by

$$S(r,\theta) = E[X^*(t,0,0)\,X(t,r,\theta)]$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{2\pi}\int_{0}^{2\pi} \exp\{i(\omega' - \omega)t + irk'\cos(\theta - \phi')\}$$
$$\cdot E[d\zeta^*(\omega,\phi)d\zeta(\omega',\phi')], \tag{19}$$

Since $X(t,r,\theta)$ is also stationary process in space, the left hand side of eq.(19), being the spatial autocovariance function, is therefore a function of r and θ only. In addition, $d\zeta^*(\omega,\phi)$ and $\zeta(\omega',\phi')$ are uncorrelated, because of the orthogonally property of $\zeta(\omega,\phi)$, so that

$$E[d\zeta^*(\omega,\phi)d\zeta(\omega',\phi')] = 0, \tag{20}$$

for all $\omega \neq \omega'$ and $\phi \neq \phi'$.

Hence, the spatial autocovariance function may be written as

$$S(r,\theta) = \int_{-\infty}^{\infty}\int_{0}^{2\pi} \exp\{irk\cos(\theta - \phi)\}E[|d\zeta(\omega,\phi)|^2]. \tag{21}$$

Substituting eq.(9) into eq.(21),

$$S(r,\theta) = \int_{-\infty}^{\infty}\left[\int_{0}^{2\pi} \exp\{irk\cos(\theta - \phi)\}h(\omega,\phi)d\phi\right]d\omega \tag{22}$$
$$= \int_{-\infty}^{\infty} g(\omega,r,\theta)d\omega, \tag{23}$$

where

$$g(\omega,r,\theta) = \int_{0}^{2\pi} \exp\{irk\cos(\theta - \phi)\}h(\omega,\phi)d\phi. \tag{24}$$

The function $g(\omega,r,\theta)$ is called a spatial covariance function at frequency ω by Henstridge (1979), which measures the covariance at frequency ω between the signals observed at the origin of a circular array and at a point on a circle (r,θ).

Equation(24) for $r = 0$ and $\theta = 0$ gives the spatial covariance function at frequency ω at the origin itself and is coincident with the spectrum of the process as

$$g(\omega, 0, 0) = \int_0^{2\pi} h(\omega, \phi) d\phi = h_0(\omega) \tag{25}$$

Setting again $r = 0$ and $\theta = 0$ in eq.(22), the spatial autocovariance function at the origin, that is, the total power of the process, is given

$$\begin{aligned} S_0 \equiv S(0,0) &= E[|X(t,0,0)|^2] \\ &= \int_{-\infty}^{\infty} \left[\int_0^{2\pi} h(\omega, \phi) d\phi \right] d\omega \\ &= \int_{-\infty}^{\infty} h_0(\omega) d\omega, \end{aligned} \tag{26}$$

which is equal to $R(0)$ in eq.(16), as is obvious from the definition.

The function $h_0(\omega)$ is called the power spectral density function of $X(t, \boldsymbol{\xi})$, (or more simply, the spectrum of $X(t, \boldsymbol{\xi})$), and it plays a fundamental role in the spectral analysis of such stationary random processes as microtremors.

We calculate the average of spatial autocovariance function with respect to azimuth at the origin of a circular array,

$$\bar{S}(r) \equiv \frac{1}{2\pi} \int_0^{2\pi} S(r, \theta) d\theta \tag{27}$$

$$\begin{aligned} &= \frac{1}{2\pi} \int_0^{2\pi} \int_{-\infty}^{\infty} g(\omega, r, \theta) d\omega d\theta \\ &= \frac{1}{2\pi} \int_0^{2\pi} \int_{-\infty}^{\infty} \int_0^{2\pi} \exp\{irk\cos(\theta - \phi)\} h(\omega, \phi) d\phi d\omega d\theta \\ &= \int_{-\infty}^{\infty} \int_0^{2\pi} h(\omega, \phi) d\omega d\phi \left[\frac{1}{2\pi} \int_0^{2\pi} \exp\{irk\cos(\theta - \phi)\} d\theta \right] \\ &= \int_{-\infty}^{\infty} \int_0^{2\pi} h(\omega, \phi) J_0(rk) d\omega d\phi \\ &= \int_{-\infty}^{\infty} J_0(rk) d\omega \int_0^{2\pi} h(\omega, \phi) d\phi, \end{aligned} \tag{28}$$

where J_0 is the Bessel function of the first kind of zero order. Then, using eq.(13) or eq.(25), we obtain

$$\bar{S}(r) = \int_{-\infty}^{\infty} h_0(\omega) J_0(rk) d\omega. \tag{29}$$

Here we define an averaged spatial covariance function at frequency ω, $\bar{g}(\omega, r)$,

$$\bar{g}(\omega, r) \equiv \frac{1}{2\pi} \int_0^{2\pi} g(\omega, r, \theta) d\theta. \tag{30}$$

Using this equation, the averaged spatial autocovariance function may be expressed as

$$\bar{S}(r) = \int_{-\infty}^{\infty} \bar{g}(\omega, r) d\omega, \tag{31}$$

and comparing this equation with eq.(29), we obtain

$$\bar{g}(\omega, r) = h_0(\omega) J_0(rk) \tag{32}$$

We define the spatial autocorrelation coefficient at frequency ω, $\rho(\omega, r)$; that is, the averaged spatial covariance function is normarized with the spectrum of the process $h_0(\omega)$, which is the same as the autocovariance function with zero time lag at a point or the spatial autocovariance function at the origin of the circular array given in eq.(13) or eq.(25).

$$\rho(\omega, r) \equiv \bar{g}(\omega, r)/h_0(\omega) \tag{33}$$

$$= J_0(rk) \tag{34}$$

In addition, eq.(33) implies that the spatial autocorrelation coefficient may be determined also on such an array that a number of seimometers are widely placed so that many sets of two seismometers with the distance r can be selected so as to cover a wide azimuthal range. The azimuthal coverage of the spatial atuocovariance function for these seismometer sets will also give the spatial autocorrelation coefficient.

Equation(33) or (34) may be expressed as

$$\rho(\omega, r) = J_0\left(\frac{r\omega}{c(\omega)}\right), \tag{35}$$

where $c(\omega)$ denotes the phase velocity of the dispersive surface waves at frequency ω.

The above theoretical consideration shows that simultaneous observations of microtremors at some points on a circle and the center of the circle for a circular array will give the phase velocity of surface waves contained in microtremors. Equation(35) means that the spatial autocorrelation coefficient of microtremors will vary in the form of the Bessel function of the first kind of zero order for different frequencies at which the phase velocities of the dispersive surface waves will be derived.

At sites for observations there may be such a condition that seismometers are set up with an improper seismometers equalization and/or a different coupling with ground surface. In order to remove this condition, an alternative expression of $\rho(\omega, r)$ may be used,

$$\hat{\rho}(\omega, r) = \frac{1}{2\pi}\int_0^{2\pi} \hat{g}(\omega, r, \theta)\left[\hat{S}(\omega, 0, 0)\hat{S}(\omega, r, \theta)\right]^{-1/2} d\theta \tag{36}$$

where, from (19) into which zero time lag at frequency ω_0 is substituted,

$$\begin{aligned} \hat{S}(\omega_0,\, r,\, \theta) &= E\left[\hat{X}^*(t,\, \omega_0,\, 0,\, 0)\hat{X}(t,\, \omega_0,\, r,\, \theta)\right], \\ \hat{S}_0(\omega_0,\, 0,\, 0) &= E\left[|\hat{X}(t,\, \omega_0,\, 0,\, 0)|^2\right], \end{aligned} \tag{37}$$

and

$$\hat{X} = aX,$$

in which $\hat{X}$ and X are the output and input microtremors respectively, and a is the amplification factor which is assumed to be independent of ω. In the derivation of (36), however, we assumed that the total energy of microtremors is the same and a constant at all sites in a circlar array, that is,

$$E\left[|\hat{X}(t,\,\omega_0,\,r,\,\theta)|^2\right] = \text{const.} \qquad \text{for all } (r,\,\theta) \tag{38}$$

Equation (36) means that the spatial autocorrelation coefficient is the azimuthal average of coherency between microtremors observed at the center of array and a site on a circle of the array.

3.2 Horizontally Polarized Waves in Microtremors[2]

So far we have considered the vertical component of waves in microtremors; that is, no polarization of waves propagating over a horizontal plane has been considered. For the horizontal component of the waves of microtremors, two types of polarization of the waves should be considered: These are the waves with vibration parallel to and those perpendicular to the direction of propagation. For those polarized waves, we assume again that those waves can be taken as a stationary random process.

To take up the polarized waves for discussion, we consider the components of the waves parallel to and perpendicular to the direction connecting two observation stations located, which will hereafter be called radial component and the tangential component of waves, respectively.

We consider again a circular array which, however, consists of 3-componet seismometers equally spaced on a circle with radius r and a 3-component seismometer at the center.

Let radial and tangential components of the waves in microtremors observed on the array be stationary random process $X_r(t,\,x,\,y)$ and $X_\theta(t,\,x,\,y)$, which inevitably include surface waves, that is, Rayleigh and Love waves. These processes may be

[2]Okada and Matsushima, 1989.

The analyses in the previous studies (Aki, 1957 and 1964; Ferrazzini et al., 1991) for horizontally polarized waves in microtremors are incorrect, because no distinction is made between the wavenumbers for Rayleigh-type and Love-type waves in microtremors.

written as,

$$
\begin{aligned}
X_r(t,\, x,\, y) &= X_r^R(t,\, x,\, y) + X_r^L(t,\, x,\, y) \\
X_\theta(t,\, x,\, y) &= X_\theta^R(t,\, x,\, y) + X_\theta^L(t,\, x,\, y)
\end{aligned}
\tag{39}
$$

where subscripts r and θ refer to radial and tangential components of waves in microtremors and superscripts R and L refer to Rayleigh- and Love-type waves.

Following the preceding sections these processes may also be written in polar form as,

$$
\begin{aligned}
X_{r,\theta}^R\,(t,\, r,\, \theta) &= \int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi} \exp\{i\omega t + irk^R\cos(\theta-\phi)\}\, dZ_{r,\theta}^R\,(\omega,\, k^R,\, \phi) \\
X_{r,\theta}^L\,(t,\, r,\, \theta) &= \int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{2\pi} \exp\{i\omega t + irk^L\cos(\theta-\phi)\}\, dZ_{r,\theta}^L\,(\omega,\, k^L,\, \phi)
\end{aligned}
\tag{40}
$$

where ω is the angular frequency, $k^R(L)$, the wavenumber for Rayleigh-type (Love-type) waves, $Z_{r,\theta}^{R(L)}$, the stationary random process for a radial or tangential component of microtremors including Rayleigh-type (Love-type) waves, and ϕ, the direction of waves coming to and through the array.

For these processes, we assume again that the spatial process Z^R and Z^L are concentrated on a curve $[\omega, k^R(\omega)]$ and on a curve $[\omega, k^L(\omega)]$, respectively . The assumption provides that the increments of the process dZ^R and dZ^L in (40) are

$$
dZ^R(\omega,\, k^R,\, \phi) = dZ^R(\omega,\, \phi), \qquad \text{and} \qquad dZ^L(\omega,\, k^L,\, \phi) = dZ^L(\omega,\, \phi) \tag{41}
$$

For the processes at the center of array and a point on a circle with radius r, we define again a spatial autocrrelation function for each component by

$$
\begin{aligned}
S_r(r,\, \theta) &= E\left[X_r^*\,(t,\, 0,\, \theta)\cdot X_r\,(t,\, r,\, \theta)\right] \\
S_\theta(r,\, \theta) &= E\left[X_\theta^*\,(t,\, 0,\, \theta)\cdot X_\theta\,(t,\, r,\, \theta)\right]
\end{aligned}
\tag{42}
$$

where E denotes the mean.

We assume that no coupling takes place between Rayleigh-type and Love-type waves in the processes; that is, $dZ^{R*}_{r,\theta}$ and $dZ^{L}_{r,\theta}$ or $dZ^{R}_{r,\theta}$ and $dZ^{L*}_{r,\theta}$ are uncorrelated so that

$$E\left[dZ^{R*}_{r,\theta}(\omega,\phi)\cdot dZ^{L}_{r,\theta}(\omega,\phi)\right]=0, \qquad \text{or} \qquad E\left[dZ^{R}_{r,\theta}(\omega,\phi)\cdot dZ^{L*}_{r,\theta}(\omega,\phi)\right]=0 \quad (43)$$

The properties from (i) to (iii) assigned in section 2 for the process Z for vertical component of waves in microtremors are also applicable to these processes. Then, eqs.(39) reduce to

$$\begin{aligned} S_r(r,\,\theta) &= E\left[X^{R*}_r(t,\,0,\,\theta)\cdot X^R_r(t,\,r,\,\theta)+X^{L*}_r(t,\,0,\,\theta)\cdot X^L_r(t,\,r,\,\theta)\right] \\ S_\theta(r,\,\theta) &= E\left[X^{R*}_\theta(t,\,0,\,\theta)\cdot X^R_\theta(t,\,r,\,\theta)+X^{L*}_\theta(t,\,0,\,\theta)\cdot X^L_\theta(t,\,r,\,\theta)\right] \end{aligned} \quad (44)$$

For the spatial autocorrelation function at frequency ω, we calculate the average with respect to azimuth at the origin of a circular array,

$$\begin{aligned} \bar{S}_r(\omega,\,r) &\equiv \frac{1}{2\pi}\int_0^{2\pi} S_r(\omega,\,r,\,\theta)d\theta \\ &= \frac{1}{2}\left[\{J_0(p)-J_2(p)\}h^R_0(\omega)+\{J_0(q)+J_2(q)\}h^L_0(\omega)\right] \end{aligned} \quad (45)$$

$$\begin{aligned} \bar{S}_\theta(\omega,\,r) &\equiv \frac{1}{2\pi}\int_0^{2\pi} S_\theta(\omega,\,r,\,\theta)d\theta \\ &= \frac{1}{2}\left[\{J_0(p)+J_2(p)\}h^R_0(\omega)+\{J_0(q)-J_2(q)\}h^L_0(\omega)\right] \end{aligned} \quad (46)$$

where J_0 and J_2 are the Bessel function of the first kind of zero order and the second order, respectively,

$$p=rk^R \qquad \textit{and} \qquad q=rk^L,$$

and h^R_0 and h^L_0 are the power spectral density functions of Rayleigh-type and Love-type waves in microtremors which can be written as

$$\begin{aligned} h^R_0(\omega)d\omega &= \frac{1}{2\pi}\int_0^{2\pi} E\left[|dZ^R_{r,\theta}(\omega,\,\phi)|^2\right] \\ &= \frac{1}{2\pi}\int_0^{2\pi} h^R_0(\omega,\,\phi)d\phi d\omega \end{aligned} \quad (47)$$

$$
\begin{aligned}
h_0^L(\omega)d\omega &= \frac{1}{2\pi}\int_0^{2\pi} E\left[|dZ_{r,\theta}^L(\omega,\,\phi)|^2\right] \\
&= \frac{1}{2\pi}\int_0^{2\pi} h_0^L(\omega,\,\phi)d\phi d\omega
\end{aligned} \tag{48}
$$

The averaged spatial autocorrelation functions for radial and tangential components at the array center are, from equations (46) and (48),

$$
\begin{aligned}
\bar{S}_r(\omega,\,0) = \bar{S}_\theta(\omega,\,0) &= \frac{1}{2}\{h_0^R(\omega) + h_0^L(\omega)\} \\
&\equiv \frac{1}{2}H_0(\omega).
\end{aligned} \tag{49}
$$

We define the spatial autocorrelation coefficient at frequency ω for radial and tangential components, $\rho_r(\omega,\,r)$ and $\rho_\theta(\omega,\,r)$ as,

$$
\begin{aligned}
\rho_r(\omega,\,r) &= \bar{S}_r(\omega,\,r)/\bar{S}_r(\omega,\,0) \\
\rho_\theta(\omega,\,r) &= \bar{S}_\theta(\omega,\,r)/\bar{S}_\theta(\omega,\,0)
\end{aligned} \tag{50}
$$

These coefficients, however, are helpfull only to understand the calculating process, and are not necessarily required to discriminate between Rayleigh-type and Love-type waves from microtremors.

Consequently, we obtain an equation for the wavenumber of Love-type waves, k_L, from equations (45) and (46),

$$
J_0(q) - \lambda J_2(q) = J_0(p) + \lambda J_2(p) \tag{51}
$$

where $J_0(p)$ and $J_2(p)$ are determined by the spatial autocorrelation function for the vertical component of waves in microtremors, if we assume that the wavenumber for the vertical component is the same as that for the horizontal component for Rayleigh-type waves. The quantity λ in (51) being a function of the frequency ω

and the radius of the circular array r, is given by the observed quantities as,

$$\lambda(\omega, r) = \frac{\Sigma^{+} - J_0(p)H_0(\omega)}{\Sigma^{-} + J_2(p)H_0(\omega)} \tag{52}$$

where

$$\Sigma^{+} = \bar{S}_r(\omega, r) + \bar{S}_\theta(\omega, r)$$
$$\Sigma^{+} = \bar{S}_r(\omega, r) - \bar{S}_\theta(\omega, r).$$

which are determined from eq.(44), the first equations of (45) and (46), and eq.(49). We can solve eq.(51) for $q(= rk^L)$ which gives the phase velocity of Love waves c^L at frequency ω.

Substituting $J_0(q)$ and $J_2(q)$ thus obtained and $J_0(p)$ and $J_2(p)$ determined for the vertical component into eqs.(45) and (46), we can also obtain the power spectral density functions of Rayleigh-type and Love-type waves at the same frequency.

4 Frequency Wavenumber Spectrun Method[3]

In the late 1960's there had been a considerable effort directed to the problem of discrimination between earthqakes and undergrond nuclear explosions on the basis of seismic data. This effort had led to sucessful results in the sense that it is now possible to discriminate between natural events and explosions for events which are above a certain magnitude threshhold. One of the factors which had led to this sucess had been the application of detection and estimation theory in seismisc problem.

Seismic waves propagated in the earth are generally corrupted by the effects of local scattering and reverberation, microtremor noise, and other factors. Thus, in order to use the recorded seismic signals for the purpose of discrimination, it is necessary to diminish the effects due to these factors. One of the most effective means for accomplishing this is through the use of an array of seismometers. Its typical example is the large aperture seismic array (LASA) in eastern Montana, USA.

The LASA has played a very important role not only in facilitating the discrimination between earthquakes and underground nuclear explosions, but also in revealing the structure of seismic noise which can be characterized as stationary random process. The latter role should be appreciated in the sense that the measurement of microtremors as being stationary random process may derive underground structures under seismic arrays.

4.1 Frequency-Wavenumber Spectral Density Function

It is well known that a stationary random process can be characterized by means of a spectral density function. Roughly speaking, this function provides the information concerning the power as a function of frequency for the stationary random process. In a similar manner, propagating waves, or a homogeneous random

[3]The following section is mainly extracted from a text book by Aki and Richards (1980).

field such as a microtremor noise, can be characterized by a frequency- wavenumber spectral density function. Loosely speaking, this function provides the information concerning the power as a function of frequency and the vector velocities of the propagating waves. Our purpose is to derive the different wave velocities and directions of approach as a function of frequency from the frequency-wavenumber spectral density function of the microtremor.

We assume again that the microtremor is stationary in both time t and two spatial coordinates x, y.

There are two basic ways of estimating the power spectrum. One is to estimate the autocovariance function and then do the Fourier transformation. The other is to directly calculate the Fourier transform of the microtremor and then do the absolute-value squaring and averaging.

These two approaches will concisely be given. Writing the microtremor as $X(x, y, t)$, the autocovariance function as $R(\xi, \eta, \tau)$, and the power spectral density as $P(k_x, k_y, \omega)$, we have

$$R(\xi, \eta, \tau) = E[X(x, y, t)\, X(x + \xi, y + \eta, t + \tau)], \tag{53}$$

and

$$P(k_x, k_y, \omega) = \int\int\int_{-\infty}^{\infty} R(\xi, \eta, \tau) \exp\{i(\omega\tau - k_x\xi - k_y\eta)\} d\tau d\xi d\eta. \tag{54}$$

In the second approach, we introduce the discrete power spectrum. We may define the power spectrum P_{lmk} by

$$P_{lmk} = \lim_{L,M,K \to \infty} \frac{E[|F_{lmk}|^2]}{L\Delta x \cdot M\Delta y \cdot K\Delta t} \tag{55}$$

where F_{lmk} is the discrete Fourier transform of the digitized microtremor $X(l\Delta x, m\Delta y, k\Delta t)$ at the points in the three-dimensional space located at an interval Δx in the x-direction, Δy in the y-direction, and Δt in the t-direction. The lengths of the data are $L\Delta x, M\Delta y$, and $K\Delta t$ in the x-, y-, and t-direction respectively.

$$F_{lmk} = \sum_{l'=0}^{L-1} \sum_{m'=0}^{M-} \sum_{k'=0}^{K-1} X(l'\Delta x, m'\Delta y, k'\Delta t)$$
$$\cdot \quad \exp\left(-i\frac{2\pi ll'}{L} - i\frac{2\pi mm'}{M} + i\frac{2\pi kk'}{K}\right) \Delta x \Delta y \Delta t. \tag{56}$$

The above methods are directly applicable to the data obtained more or less continuously in space. When the microtremor is completely stationary, a mobile seismometer can be used repeatedly to cover any desired spatial point, making the measurement of the autocovariance function continuous in space.

In most cases, however, the microtremor is not completely stationary, and is caused by atmospheric and oceanographic disturbances, which are transient. The seismic arrays designed for the study of noise or microtremor are usually immobile and make the continuous spatial coverage of the autocovariance function difficult. Thus the basic methods described earlier cannot apply to most data.

4.2 Beam-Forming Method (BFM)

In practice, several approximate methods for estimating frequency-wavenumber spectrum have been developed. The simplest methed is to combine beam-forming with a power-spectrum estimate for the beam output. This method is called a conventional method as well. The time shift required for beam-forming for the point (k_x, k_y, ω) is

$$t_i = t_0 + \frac{k_x}{\omega}(x_i - x_0) + \frac{k_y}{\omega}(y_i - y_0) + \tau_i, \tag{57}$$

where t_0 is the arrival time at a reference point (x_0, y_0) and τ_i is the station residual ($-\tau_i$ is the station correction). Expressing the microtremor time-series at the i-th station as $X_i(t)$, the beam output can be written as

$$b(k_x/\omega, k_y/\omega, t) = \frac{1}{N}\sum_{i=1}^{N} X_i(t + t_i). \tag{58}$$

The power spectrum of $b(k_x/\omega, k_y/\omega, t)$ as a time series can be obtained by calculating the autocovariance and then performing the Fourier transform. The result is

$$\hat{P}(k_x, k_y, \omega) = \int \exp(i\omega\tau)d\tau \frac{1}{N^2} E\left[\sum_{i=1}^{N} X_i(t+t_i) \sum_{j=1}^{N} X_j(t+\tau+t_j)\right]. \tag{59}$$

By definition eq.(53), we can write this result as

$$\hat{P}(k_x, k_y, \omega) = \int \exp(i\omega\tau)d\tau \frac{1}{N^2} \sum_{i,j=1}^{N} R(x_j - x_i, y_j - y_i, t_j - t_i + \tau). \tag{60}$$

Introducing a weight function

$$W(\kappa_x, \kappa_y) = \frac{1}{N^2} \sum_{i,j=1}^{N} \exp[-i\kappa_x(x_i - x_j) - i\kappa_y(y_i - y_j) + i\omega(\tau_i - \tau_j)], \tag{61}$$

we shall show that our simple estimate $\hat{P}$ is a weight average of the true power spectrum according to the formula

$$\hat{P}(k_x, k_y, \omega) = \iint_{-\infty}^{\infty} W(\kappa_x - k_x, \kappa_y - k_y) P(k_x, k_y, \omega)\, d\kappa_x d\kappa_y. \tag{62}$$

Inserting eq.(61) into eq.(62), we find

$$\begin{aligned} \hat{P}(k_x, k_y, \omega) \;=\; & \sum_{i,j} \iint_{-\infty}^{\infty} \frac{1}{N^2} \exp[i\kappa_x(x_j - x_i) + i\kappa_y(y_j - y_i)] \\ & \cdot \exp[(-ik_x(x_j - x_i) - ik_y(y_j - y_i) - i\omega(\tau_j - \tau_i)] \\ & \cdot P(\kappa_x, \kappa_y, \omega)\, d\kappa_x d\kappa_y. \end{aligned} \tag{63}$$

On the other hand, from the inverse transform of eq.(54),

$$R(x, y, \tau) = \frac{1}{8\pi^3} \iiint_{-\infty}^{\infty} P(\kappa_x, \kappa_y, \omega) \exp(-i\omega\tau + i\kappa_x x + i\kappa_y y) d\omega d\kappa_x d\kappa_y \tag{64}$$

or

$$\int_{-\infty}^{\infty} R(x,y,\tau') \exp(i\omega\tau')\, d\tau' = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} P(\kappa_x,\kappa_y,\omega) \exp(i\kappa_x x + i\kappa_y y)\, d\kappa_x d\kappa_y. \tag{65}$$

Putting this result into eq.(63), we have

$$\begin{aligned} \hat{P}(k_x,k_y,\omega) &= \frac{1}{N^2}\sum_{i,j}\int_{-\infty}^{\infty} \exp(i\omega\tau')\, d\tau' R(x_j-x_i, y_j-y_i, \tau') \\ &\quad \cdot \exp[-ik_x(x_j-x_i) - ik_y(y_j-y_i) - i\omega(\tau_j-\tau_i)]. \end{aligned} \tag{66}$$

Rewriting

$$\tau = \tau' - \frac{k_x}{\omega}(x_j-x_i) - \frac{k_y}{\omega}(y_j-y_i) - (\tau_j-\tau_i), \tag{67}$$

we obtain

$$\hat{P}(k_x,k_y,\omega) = \frac{1}{N^2}\sum_{i,j}\int_{-\infty}^{\infty} \exp(i\omega\tau) d\tau R(x_j-x_i, y_j-y_i, \tau + t_j - t_i). \tag{68}$$

which agrees with eq.(60). Therefore, the power spectrum of the beam output is a weight average of the true power spectrum. The weight function $W(\kappa_x,\kappa_y)$ can be calculated by eq.(61) once the station distribution (x_i, y_i) is known.

Two subsets of LASA seismographs and their corresponding weight functions are shown in Fig.1.

If the weight function is a delta function centered at $k_x = k_y = 0$, our estimate gives the exact value of the true spectrum. The figure, however, shows a spread (6 dB down from the peak at the center) of about ± 0.035 km^{-1} for the array of diameter 22 km, and about ± 0.025 km^{-1} for the array of diameter 30 km. Examples of actual wavenumber spectra for various frequencies are shown in Fig.2. LaCoss et al.(1969) studied the mode structure of seismic noise recorded at LASA and found that noise at frequencies higher than 0.3 Hz is primarily compressional waves that probably originate beneath large storms at sea; the noisiest band between 0.2 to 0.3 Hz consists of both body waves and higher-mode Rayleigh waves. At frequencies lower than 0.15 Hz, vertical-component microtremors consist primarily of fundamental-mode Rayleigh waves.

4.3 High-Resolution Method

Another method, called the maximum-likelihood estimator, developed by Capon (1969), is also claimed to have a higher resolution than the conventional method. In order to understand this method, we assume that data $d_{t,i}$ with a finite length N obey the Gaussian distribution with the mean value s_t and the covariance matrix, $\boldsymbol{\rho}$. The probability density function for $M * N$ variables $d_{t,i}$ can be written as

$$f = \frac{|\Phi|^{1/2}}{(2\pi)^{MN/2}} \exp\left\{ -\frac{1}{2} \sum_{i,j=1}^{M} \sum_{k,l=1}^{N} \Phi_{ij}^{kl}(d_{k,i} - s_k)(d_{l,j} - s_l) \right\}, \tag{69}$$

where Φ_{ij}^{kl} is an element of the $MN * MN$ matrix Φ that is the inverse matrix of covariance matrix, whose element is

$$\rho_{ij}^{kl} = E[(d_{k,i} - s_k)(d_{l,j} - s_l)], \tag{70}$$

Subscripts i and j refer to station, and superscripts k and l refer to times.

Here we consider the simple case of one station ($M = 1$) in eq.(69). The probability density function for N variables d_t can be written as

$$f = \frac{|\Phi|^{1/2}}{(2\pi)^{N/2}} \exp\left\{ -\frac{1}{2} \sum_{k,l=1}^{N} \Phi^{kl}(d_k - s_k)(d_l - s_l) \right\}, \tag{71}$$

where Φ^{kl} is an element of $N * N$ matrix Φ, which is the inverse matrix of the covariance matrix $\boldsymbol{\rho}$ whose element is

$$\rho^{kl} = E[(d_k - s_k)(d_l - s_l)]. \tag{72}$$

We assume that the signal s_k has the known shape $f_k (k = 1, 2, ..., N)$ but that its amplitude contains an unknown factor c :

$$s_k = cf_k \tag{73}$$

We want the maximum-likelihood estimate of c. Using the matrix notation for brevity, the exponent in eq.(71) can be written as $-\frac{1}{2}(\mathbf{d} - c\mathbf{f})^T\mathbf{\Phi}(\mathbf{d} - c\mathbf{f})$ where $\mathbf{d}$ and $\mathbf{f}$ are column vectors with the components d_k and f_k, respectively, and ^{T} indicates taking the transpose of the vector.

$$-\frac{1}{2}(\mathbf{d} - c\mathbf{f})^T\mathbf{\Phi}(\mathbf{d} - c\mathbf{f}) = -\frac{1}{2}\left[\mathbf{d}^T\mathbf{\Phi}\,\mathbf{d} - c\,\mathbf{d}^T\mathbf{\Phi}\,\mathbf{f} - c\mathbf{f}^T\mathbf{\Phi}\,\mathbf{d} + c^2\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f}\right]. \tag{74}$$

Taking the derivative with respect to c , setting the result equal to zero, and observing $\mathbf{d}^T\mathbf{\Phi}\,\mathbf{f} = \mathbf{f}^T\mathbf{\Phi}\,\mathbf{d}$ because $\mathbf{\Phi}$ is symmetric, we find the maximum likelihood estimate to be

$$\hat{c} = \frac{\mathbf{d}^T\mathbf{\Phi}\,\mathbf{f}}{\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f}} \tag{75}$$

The corresponding estimate of $\mathbf{s} = c\mathbf{f}$ is

$$\hat{\mathbf{s}} = \hat{c}\mathbf{f} = \frac{\mathbf{d}^T\mathbf{\Phi}\,\mathbf{f}}{\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f}}\mathbf{f}, \tag{76}$$

which is equal to $c\mathbf{f}$ when $\mathbf{d}$ is equal to $c\mathbf{f}$. In other words, it does not distort the signal. We can find the variance of the estimate c . Rewriting

$$\hat{c} - c = (\mathbf{d} - c\mathbf{f})^T\mathbf{\Phi}\,\mathbf{f}(\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1}, \tag{77}$$

and noting that

$$E[(\mathbf{d} - c\mathbf{f})(\mathbf{d} - c\mathbf{f})^{-1}] = \rho \qquad \text{and} \qquad \rho = \mathbf{\Phi}^{-1}, \tag{78}$$

we obtain

$$\begin{aligned}
E[(\hat{c}-c)^2] &= E[(\hat{c}-c)^T(\hat{c}-c)] \\
&= (\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1}\mathbf{f}^T\mathbf{\Phi}\cdot E\left[(\mathbf{d}-c\mathbf{f})(\mathbf{d}-c\mathbf{f})^T\right]\,\mathbf{\Phi}\,\mathbf{f}\,(\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1} \\
&= (\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1}\mathbf{f}^T\mathbf{\Phi}\,\rho\,\mathbf{\Phi}\,\mathbf{f}\,(\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1} \\
&= (\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1}\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f}(\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1} \\
&= (\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1} \\
&= (\mathbf{f}\rho^{-1}\mathbf{f})^{-1}.
\end{aligned} \tag{79}$$

Summarizing the above result, the maximum likelihood estimate of the signal amplitude c is given by $(\mathbf{d}^T\mathbf{\Phi}\,\mathbf{f})(\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f})^{-1}$, and the variance of the estimate is equal to $(\mathbf{f}^T\boldsymbol{\rho}^{-1}\,\mathbf{f})^{-1}$, where $\boldsymbol{\rho}$ is the covariance matrix of noise and $\mathbf{\Phi} = \boldsymbol{\rho}^{-1}$.

The maximum-liklihood estimate of the power spectrum used by Capon(1969) is the variance of the signal estimate when the signal shape is the unit sinusoidal oscillation: $f_k = \exp[i\omega(k-l)\Delta t]$, where ω is the frequency at which the power spectrum is to be estimated. The power spectral estimate is

$$\frac{1}{\mathbf{f}^T\mathbf{\Phi}\,\mathbf{f}^*} = \frac{1}{\sum_{k=1}^{N}\sum_{l=1}^{N}\Phi^{kl}\exp[i\omega(k-l)\Delta t]} \tag{80}$$

where a conjugate operation * is included because $\mathbf{f}$ is complex. Equation (80) is a reasonable estimate of the power spectrum, because it is the variance of the best estimate of a virtual sinusoid with a given frequency. Since the variance is caused by the noise power in the vicinity of that frequency, this must give a high resolution estimate of the noise power spectrum at that frequency.

Capon's estimate of the frequency-wavenumber spectrum is given by a natural extension of eq.(80) to the two-dimensional case:

$$P(k_x, k_y, \omega) = \left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} \phi_{ij}(\omega) \exp[ik_x(x_i - x_j) + ik_y(y_i - y_j)] \right\}^{-1}, \tag{81}$$

where $\phi_{ij}(\omega)$ is an element of the matrix $\phi(\omega)$; (x_i, y_i) indicates the i-th seismometer location. $\phi(\omega)$ is the inverse matrix of the Fourier transform of the covariance matrix $\rho_{\tau,ij}$, given by

$$\rho_{\tau,ij} = E[n_{t,i} n_{t+\tau,j}], \tag{82}$$

where $n_{t,i}$ is the noise at the i-th station.

We have extensively used this method[4] in estimating underground structures at many sites in the Tokachi Basin in eastern Hokkaido, Japan, using long-period microtremors. Depths of underground structures estimated reach to 2,000 - 3,000 m in the period range of 1 to 3.5 s of the fundamental-mode Rayleigh waves in microtremors. Examples of the results obtained will be presented in the lecture, in which some fundamental research on the method also will be given.

[4]Recently, Miyakoshi et al.(1996) have pointed out through a numerical simulation test for the Capon's estimate of the frequency-wavenumber spectrum that the estimate has a disadvantage that inaccurate phase velocities may be given, since the degeneration of the plural maxima in the frequency-wavenumber spectra sometimes takes place at lower frequencies. This may be intrinsic to the spatial extent of array employed where a small number of seismometers are used.

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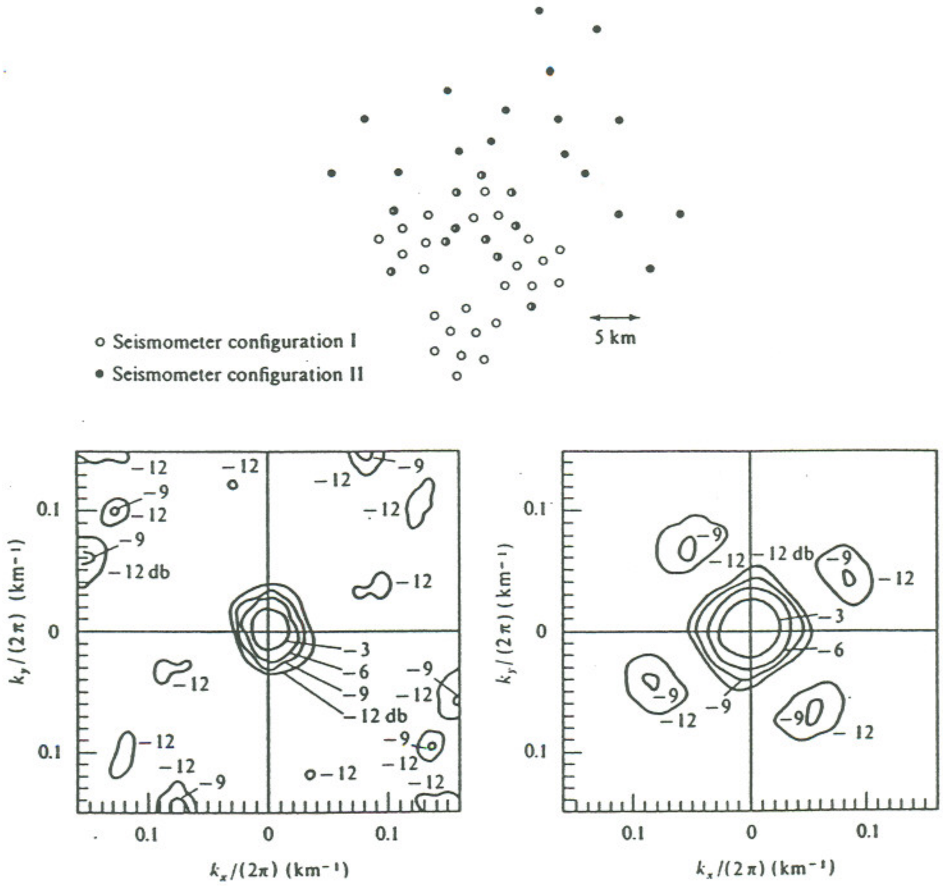

FIGURE 1

Examples of the weight function $W(k_x, k_y)$ for two different array configurations; configuration I (array diameter 22 km) is on the right; configuration II (diameter 30 km), on the left. [From LaCoss et al., 1969.]

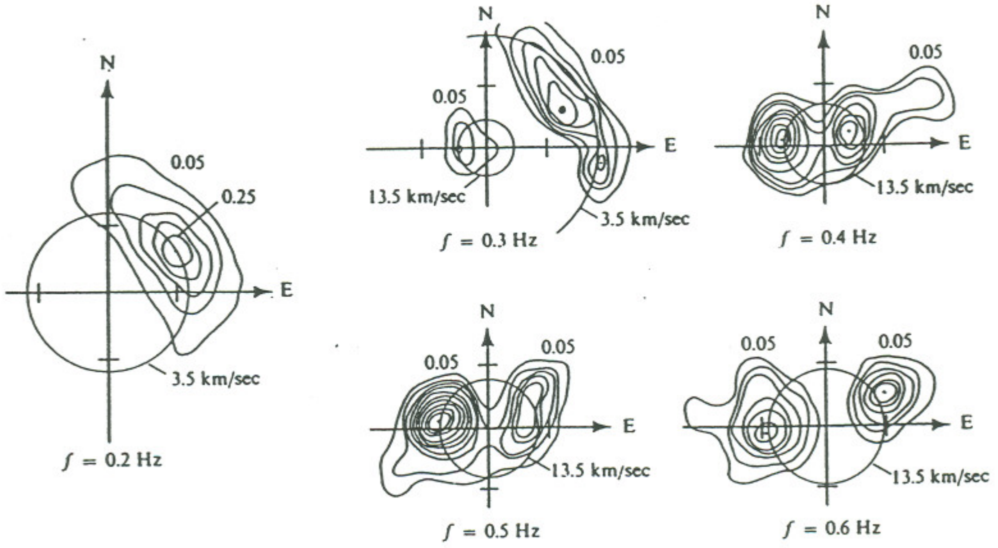

FIGURE 2

Wavenumber spectra of microseisms for five different frequencies observed at the Montana LASA. [From LaCross et al., 1969.]